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Edited by G. V. E. THOMPSON

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Our Society

We wish to draw members' attention to three changes introduced this month on the page giving particulars of the Society and its Officers, changes which we are sure will be warmly welcomed by all.

Firstly, the offices of President and Vice-President replace those of Chairman and Vice-Chairman of Council. Secondly, the Scottish and Yorkshire Provisional Branches have been granted full Branch status. Both changes are referred to more fully in the News Section, on p. 387.

Finally, the following statements have been introduced to amplify the title of the Society:

Society for the Promotion of the Science and Technology of Aerospace and Astronautics

A Founder Member-Society of the International Astronautical Federation

The first subtitle indicates that the Society is concerned with both scientific and engineering aspects of our field, and with both the atmospheric fringe (aerospace) and outer space itself. It should not be taken as implying that we are uninterested in the legal, economic and sociological problems involved—perusal of past issues of the *Journal* will show that this is not the case. The second statement indicates our participation in the work of the appropriate international scientific union—the I.A.F. The Society played a leading role in the formation of the I.A.F., drew up its original draft Constitution and has since been a major contributor to its work and finance.

No change in the Society's policy is implied by the first subtitle. Neither is it the prelude to a change in name. From time to time we receive suggestions for changes, including the dropping of the word *British*, or the changing of the word *Interplanetary* to *Rocket*, *Aerospace*, *Astronautics*, *Space*, etc. However, the title of the Society was chosen with care and will be of value for many years yet—it would remain valid even were interstellar travel to become a reality. It is unnecessary for us to keep changing our name to accord with the current catchphrase of the moment, and our interests are not restricted to a particular form of propulsion—indeed, propulsion is but one of the many subjects we study.

The need for retaining the word *British* seems obvious, for ours is a Society founded, incorporated and having its headquarters in the United Kingdom and although we have a large international membership we do not wish to encroach, or even appear to encroach, upon the territories of other astronautical societies.

A word on our relation to other British societies is not out of place here. We are not disturbed to their interesting themselves in appropriate aspects of space-flight. On the contrary, we welcome and encourage such interest, and the series of joint symposia which we have arranged has this as one of its objects. By arousing interest elsewhere we are promoting the science and technology of astronautics. Even where the Society does not directly participate in meetings of other bodies, we often assist them by providing or suggesting speakers.

There is no need to draw up lines of demarcation, for the activities of various societies must inevitably overlap. Thus, although this Society is not concerned with aircraft as such, we cannot avoid dealing with winged re-entry vehicles. Similarly, although it would be inappropriate for the Royal Aeronautical Society to spend the majority of its time discussing interplanetary travel, it quite rightly covers such topics as guided missiles, the use of rockets for assisted take-off, and aerospace activities.

Covering as we do all possible extra-terrestrial actions of man and his instruments (plus terrestrial construction, launching and support facilities) it is inevitable that our interests will have some common ground with those of many other organizations. However, ours is the only society in Britian dealing in the broadest sense with all aspects of aerospace and astronautics.

In disccusing the astronautical interests of other professional societies it is appropriate to mention the Royal Society. This eminent body has a unique place in the history of science, being the oldest and most distinguished of all learned societies. Now, in its tercentenary year, by its appreciation of the vast possibilities of space research, it has demonstrated that it is still in the forefront of scientific thought. The B.I.S. hopes that the United Kingdom Government will display the same degree of enlightenment in taking the necessary steps to ensure that the nation shall play an active role in exploring this great new frontier.

Recovery from Orbit

We offer our congratulations to the members of the American and Russian teams whose work on the Discovery and 'Arknik' satellites respectively has been crowned with success. These first recoveries from orbit are great technical achievements.

As we go to press little information has been made available about the Russian spaceship. However, it seems evident that it will not be long before man himself will orbit the globe and re-enter the atmosphere. Indeed we would not be surprised if one of the Russian delegates to the 1961 International Astronautical Congress were able to report his personal observations of the other side of the Moon.

G. V. E. THOMPSON

GRAPHICAL PLOTTING OF INTERPLANETARY ORBITS*

By M. VERTREGT,† Fellow

ABSTRACT

A method is described in which a conic section can be represented by straight lines; it enables a point on a conic section to be constructed easily if the length of the radius vector, or the angular pericentre distance is given.

Methods are given for the construction of tangents to conic sections, for the graphical construction of the time needed for a body to move from one point to another along an elliptical orbit, and for the plotting of an interplanetary orbit and of the direction and magnitude of the transfer impulses.

I. THE BASIS OF THE CONSTRUCTION

FROM an astronautical standpoint, most graphical constructions of conic sections have the disadvantage that they start from the centre, whereas what we need is a focal construction. Moreover, it is mostly not possible to construct a special point, *e.g.*, a point with a given radius-vector or with a given pericentre-distance. The usual procedure is to construct several points, to connect these with a smooth line and thus to find the desired point indirectly.

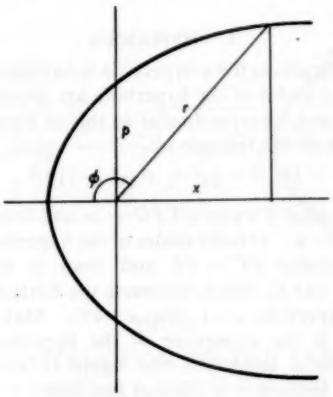

FIG. 1.

The focal equation for a conic in polar co-ordinates (Fig. 1) is:

$$r = p/(1 + \epsilon \cos \phi) \quad .. \quad .. \quad (1)$$

We have:

$$\cos \phi = - x/r$$

from which:

$$r = p + \epsilon x \quad .. \quad .. \quad .. \quad (2)$$

This equation suggests a simple construction.

Let us assume that the parameter p and the eccentricity ϵ of a conic section are given (Fig. 2). Then, from a given point F, the focal point, we erect a perpendicular FH with length p.

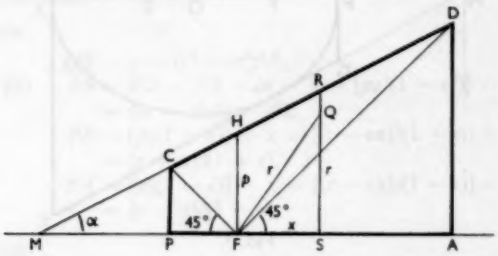

FIG. 2.

We draw a line MHD in such a manner that $\tan \alpha = \epsilon$. Then every perpendicular RS with distance x from F will have a length r in accordance with Equation (2).

Now, if we trace a circle with centre F and radius r, which intersects RS in Q, then Q is a point of the ellipse. To complete the diagram we draw two lines FC and FD which make angles of 45° with the base MA. From points C and D we drop perpendiculars CP and DA on the axis MA.

Now if

$$CP = PF = - x_1$$

Equation (2) gives

$$x_1 = - p/(1 + \epsilon).$$

But this is the pericentre-radius of the ellipse. Therefore point P is the pericentre.

Similarly, $DA \equiv x_2 = p/(1 - \epsilon)$, which is the apocentre-radius, and therefore A is the apocentre of the ellipse.

If we construct the points G and E, symmetrically to the major axis PA (Fig. 3), we get a trapezium, which we can consider as a transformation of the enclosed ellipse, because it shows many characteristics of the ellipse. These are:

(1) The line FH is the parameter p of the ellipse.
(2) The distance FP is the pericentre-radius of the ellipse.
(3) The distance FA is the apocentre-radius of the ellipse.
(4) PA is major axis of the ellipse.
(5) OX is the semi-major axis of the ellipse (O is the centre).

* Manuscript received 29 May, 1959.

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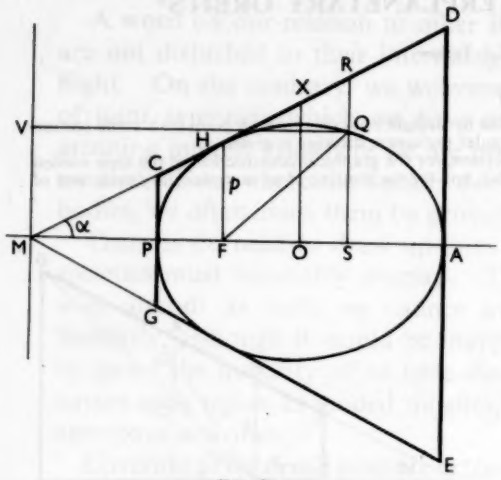

FIG. 3.

(6) The slope of the line CD, or $\tan\alpha$, is equal to the eccentricity ϵ of the ellipse.

(7) The perpendicular MV in M is the directrix of the ellipse. For if MV is the directrix and $VQ \parallel MS$, then according to a well-known definition of a conic section: $FQ = RS = MS \tan\alpha = \epsilon\, VQ$.

II. CONSTRUCTIONS FOR ELLIPSES, PARABOLAE AND HYPERBOLAE

Elements of a conic section are:

(1) The semi-major axis a.
(2) The semi-minor axis b.
(3) The parameter p.
(4) The eccentricity ϵ.

These elements are connected by the relations:

$$b = a\sqrt{(1-\epsilon^2)},$$
$$p = a(1-\epsilon^2).$$

If two of these four elements are known, the trapezium $CDEG$ can be constructed.

1. ELLIPSE

If the length of a radius r of an ellipse is given (AK, Fig. 4), we draw $RK \parallel PA$, draw $RS \perp PA$, and $FQ = RS$, then Q is a point of the ellipse and ϕ is the pericentre-angle of the radius-vector.

Alternately, if the angle ϕ is given we proceed as follows (Fig. 5). Draw a line FL, which makes an angle ϕ with PA, set off $AN = FL$. Draw FN, which intersects CD in R. Draw $RS \perp PA$, which intersects FL in Q. Then Q is a point of the ellipse and the length of the radius-vector is $FQ = RS$.

2. PARABOLA

For a parabola (Fig. 6), we have to know only the parameter p. Therefore we erect a perpendicular FH with the given length p on MS. We circle $MF = FH$ and draw the line MR. Bisect MF and draw a perpendicular PC in P. Then P is the pericentre of the parabola. As the eccentricity of a parabola is 1, then $\alpha = 45°$.

3. HYPERBOLA

The construction for a hyperbola is as follows (Fig. 7): If the axis a and b of the hyperbola are given, we set off a length a and a perpendicular to this of length b. The hypotenuse of the triangle is:

$$(a^2 + b^2)^{1/2} = [a^2 + a^2(\epsilon^2 - 1)]^{1/2} = a\epsilon.$$

Now from point F we set off $FO = a\epsilon$ and from O, $OP = a$, and $OZ = a$. O is the centre of the hyperbola. Draw a perpendicular $PC = PF$ and draw a straight line through Z and C, which intersects the horizontal in M.

For a hyperbola, $\epsilon > 1$, thus $\alpha > 45°$. Make $PW = b$, than OW is the asymptote of the hyperbola. For a parabola or a hyperbola the figure is not a proper trapezium, because it is open at one side.

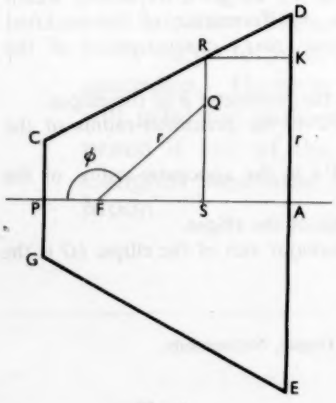

Fig. 4

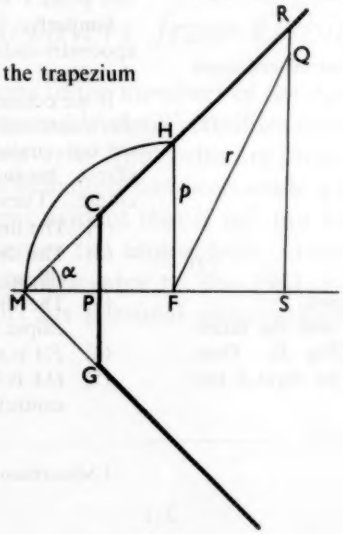

FIG. 5.

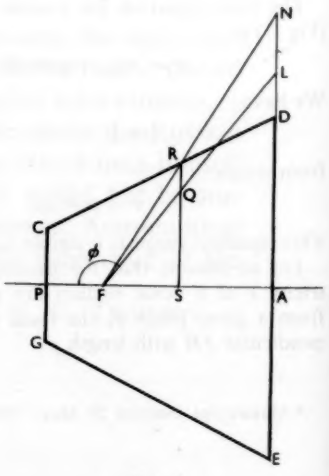

FIG. 6.

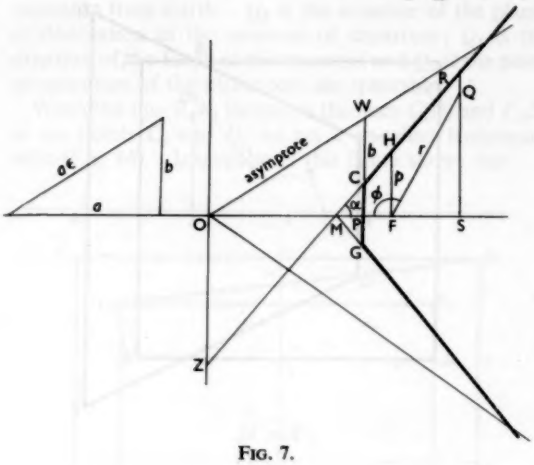

FIG. 7.

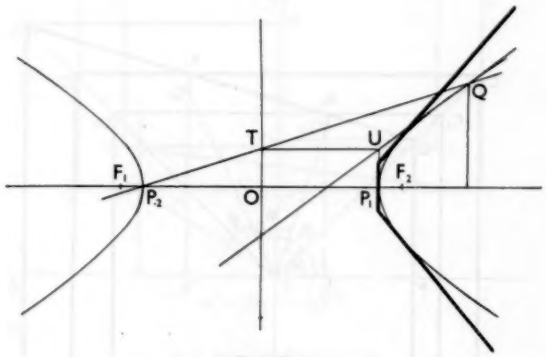

FIG. 7a.

III. CONSTRUCTION OF THE TANGENT

To draw a tangent in a point Q of a conic section we first derive the equation of the tangent (Fig. 8).

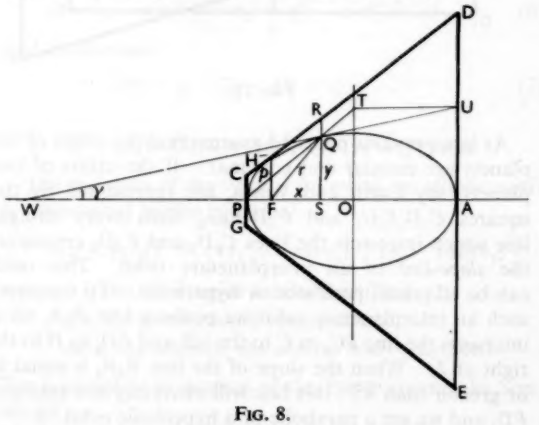

FIG. 8.

Since
$$y = (r^2 - x^2)^{1/2} = [(p + \epsilon x)^2 - x^2]^{1/2}$$

then:
$$\frac{dy}{dx} = \frac{[\epsilon(p + \epsilon x) - x]}{[(p + \epsilon x)^2 - x^2]^{1/2}} = \frac{(r\epsilon - x)}{(r^2 - x^2)^{1/2}} = \tan\gamma \quad .. \quad (4)$$

We now construct the tangent as follows:

Draw a line through P and Q which intersects the perpendicular on O in T. Draw $TU \| PA$. Then WQU is the tangent in Q.

This can be demonstrated as follows:

$TO = QS.PO/PS$, and $WS = SA.QS/(TO - QS)$,
$$\tan\gamma = QS/WS = (TO - QS)/SA = [(QS.PO/PS) - QS]/SA = QS\,(PO - PS)/(PS.SA).$$

Now:
$$QS = y = (r^2 - x^2)^{1/2},$$
$$PO - PS = FO - FS = a\epsilon - x = [p\epsilon/(1 - \epsilon^2)] - x = (r\epsilon - x)/(1 - \epsilon^2),$$
$$PS = [p/(1 + \epsilon)] + x = [(r - \epsilon x)/(1 + \epsilon)] + x = (r + x)/(1 + \epsilon),$$
$$SA = [p/(1 - \epsilon)] - x = [(r - \epsilon x)/(1 - \epsilon)] - x = (r - x)/(1 - \epsilon).$$

Thus:
$$\tan\gamma = (r^2 - x^2)^{1/2} \cdot \frac{r\epsilon - x}{1 - \epsilon^2} \Big/ \left[\frac{r + x}{1 + \epsilon} \cdot \frac{r - x}{1 - \epsilon}\right] = (r\epsilon - x)/(r^2 - x^2)^{1/2}.$$

in accordance with Equation (4).

For point H, $x = O$, and $\tan\gamma = \epsilon$. Thus the line CD is a tangent to the ellipse in point H.

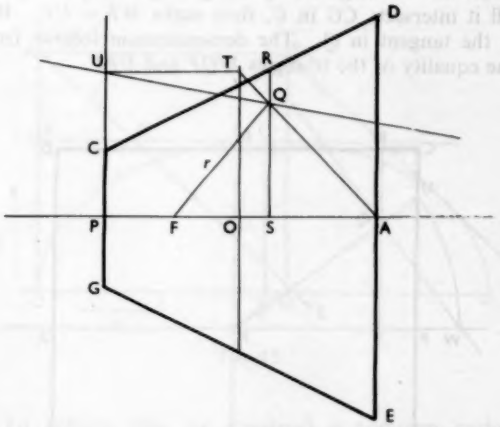

FIG. 9.

Fig. 9 gives the construction of the tangent at point Q of an ellipse on the other side of the centre. A similar construction can be used for a hyperbola. A hyperbola has two branches and the centre O (Fig. 7*a*) is situated in the middle of the line connecting the two foci. Call the pericentre of the first branch P_1, and that of the second branch P_2. Draw a line from Q to P_2, which intersects the perpendicular through O in a point T. From T draw a line parallel to the major axis, which intersects a perpendicular through P_1 in a point U. Draw a line through Q and U; this line is the tangent in Q.

For a parabola, $\epsilon = 1$, and thus according to Equation (4):

$$QS/(FW + SF) = \tan\gamma = (r - x)/(r^2 - x^2)^{1/2}$$
$$= (r^2 - x^2)^{1/2}/(r + x) = y/(r + x).$$

As $SF = x$, FW must be equal to r. Therefore the construction of the tangent is as follows (Fig. 10): Make

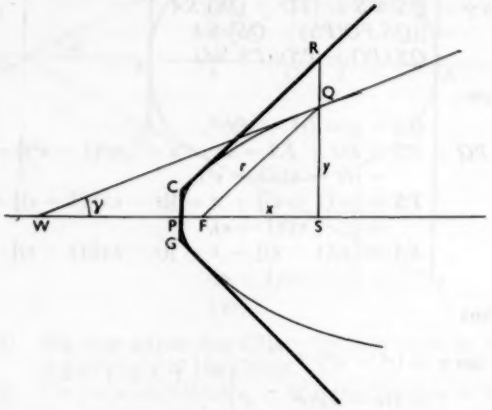

FIG. 10.

$FW = r$ and join W with Q, then WQ is the tangent in Q.

In this system a circle is represented by a square and its radius is, of course, half the side of the square.

To draw a tangent at point Q (Fig. 11), prolong FQ till it intersects CG in U, then make $WF = UF$. WQ is the tangent in Q. The demonstration follows from the equality of the triangles WQF and UPF.

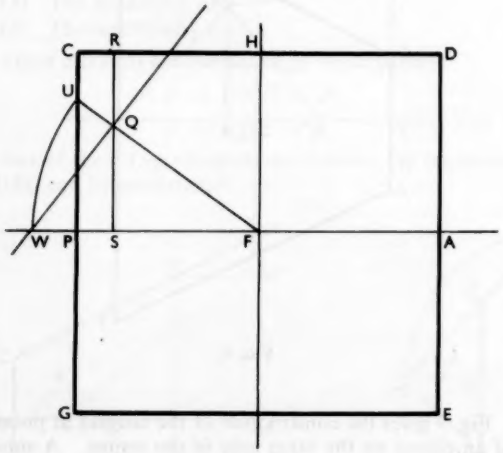

FIG. 11.

IV. INTERPLANETARY ORBITS

The intersection of an ellipse and a circle is represented in Fig. 12, and the intersection of an ellipse with two concentric circles in Fig. 13. This is the case of an interplanetary orbit.

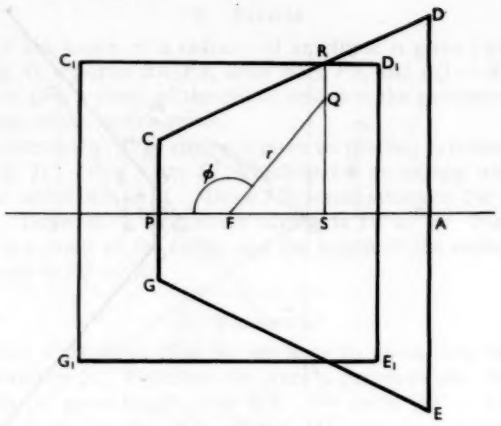

FIG. 12.

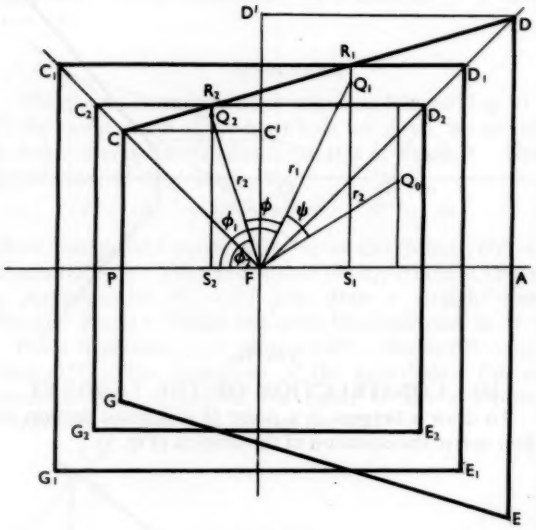

FIG. 13.

As in a previous paper,[1] I assume that the orbits of the planets are circular and coplanar. If the orbits of two planets, say Earth and Venus, are represented by the squares $C_1D_1E_1G_1$ and $C_2D_2E_2G_2$, then every straight line which intersects the lines C_1D_1 and C_2D_2 represents the *slope-line* of an interplanetary orbit. This orbit can be elliptical, parabolic or hyperbolic. To construct such an interplanetary orbit we prolong line R_2R_1 till it intersects the line FC_1 in C to the left and FD_1 in D to the right of F. When the slope of the line R_2R_1 is equal to or greater than 45°, this line will obviously not intersect FD_1 and we get a parabolic or a hyperbolic orbit.

Construction of the two points of intersection Q_1 and Q_2 of the ellipse with the two circles gives us the angle ϕ or the angular distance of the voyage. ψ is the angular distance of the planets at the moment of departure of the

spaceship from Earth. Q_0 is the situation of the planet of destination at the moment of departure; Q_1 is the situation of the Earth at this moment and Q_2 is the point of encounter of the planet and the spaceship.

When the line R_2R_1 intersects the lines C_1D_1 and C_2D_2 in the points C_2 and D_1 we get a so-called Hohmann-orbit (Fig. 14). Inspection of this figure shows that:

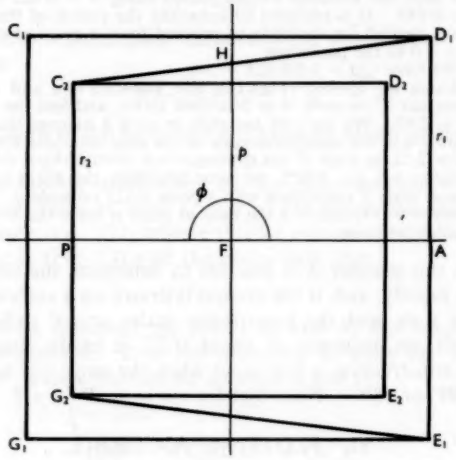

FIG. 14.

$$a = (r_1 + r_2)/2.$$

If:

$$q = a/r_1 \text{ and } n = r_2/r_1$$

then:

$$q = (1 + n)/2 \quad .. \quad .. \quad .. \quad (5)$$

and:

$$\epsilon = (r_1 - r_2)/(r_1 + r_2) = (1 - n)/(1 + n) \quad .. \quad (6)$$

and furthermore:

$$FH = p = 2n\,r_1/(1 + n) \quad .. \quad .. \quad (7)$$

V. THE DURATION OF A VOYAGE

The duration of the voyage following an elliptic orbit is represented by the equation[1]:

$$T = a^{3/2}.(fM)^{-1/2}.\left\{(\tau_1 - \epsilon \sin \tau_1) - (\tau_2 - \epsilon \sin \tau_2)\right\},$$

where a is the semi major axis of the ellipse,

f is the gravitational constant, and

M is the mass of the central body (the Sun).

For a complete revolution of the Earth around the Sun, we get:

$$T' (= 1 \text{ year}) = r^{3/2}.(fM)^{-1/2}\,2\pi,$$

where r is the radius of the Earth's orbit, and,

M is the mass of the Sun.

Then

$$T \text{ (in years)} = a^{3/2}.(fM)^{-1/2}.\left\{(\tau_1 - \epsilon \sin \tau_1) - (\tau_2 - \epsilon \sin \tau_2)\right\}/r^{3/2}.(fM)^{-1/2}.2\pi$$

$$= (q^{3/2}/2\pi).\left\{(\tau_1 - \epsilon \sin \tau_1) - (\tau_2 - \epsilon \sin \tau_2)\right\} \quad (8)$$

where:

q is the semi major axis of the elliptic orbit in A.U.

$$\tau_1 = \text{arc cos } [(1 - r_1/a)/\epsilon] \quad .. \quad .. \quad (9)$$

$$\tau_2 = \text{arc cos } [(1 - r_2/a)/\epsilon] \quad .. \quad .. \quad (10)$$

where r_1 and r_2 are radii of the ellipse and a is the semi major axis.

If we call:

$$\tau_1 - \epsilon \sin \tau_1 = T_1' \quad .. \quad .. \quad .. \quad (11)$$

and:

$$\tau_2 - \epsilon \sin \tau_2 = T_2' \quad .. \quad .. \quad .. \quad (12)$$

then Equation (8) becomes:

$$T = (q^{3/2}/2\pi)(T_1' - T_2) \quad .. \quad .. \quad (13)$$

The calculation of this form is rather cumbersome and a graphical construction will save much time, if the required accuracy is not too high.

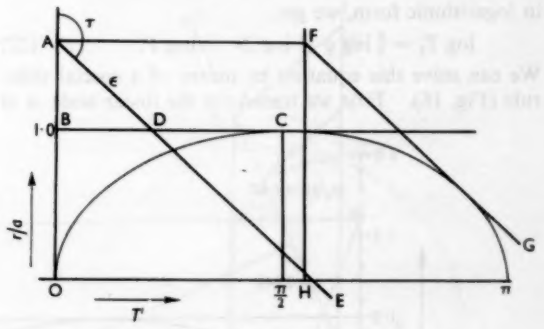

FIG. 15.

To achieve this, we construct a common cycloid (Fig. 15) of which the equations are:

$$x = \tfrac{1}{2}(2\tau - \sin 2\tau) \quad .. \quad .. \quad (14)$$

$$y = \tfrac{1}{2}(1 - \cos 2\tau) \quad .. \quad .. \quad .. \quad (15)$$

and:

$$\frac{dy}{dx} = \cot \tau \quad .. \quad .. \quad .. \quad (16)$$

The equation of a tangent to the cycloid is:

$$y' - y = \frac{dy}{dx}(x' - x) \quad .. \quad .. \quad (17)$$

where y' and x' are the co-ordinates of the tangent and y and x the co-ordinates of the tangential point.

Introducing Equations (14), (15) and (16) into Equation (17) we get:

$$x' = [(y' - 1)/\cot\tau] + \tau \quad .. \quad .. \quad (18)$$

We now make:

$$y' = r/a \quad .. \quad .. \quad .. \quad (19)$$

and construct:

$$\tau = \text{arc cos } [(1 - r/a)/\epsilon] \quad .. \quad .. \quad (20)$$

which gives:

$$r/a = 1 - \epsilon \cos\tau$$

Then:

$$y' = 1 - \epsilon \cos\tau$$

Substituting this in Equation (18) we get:

$$x' = -[\epsilon \cos\tau/\cot\tau] + \tau = \tau - \epsilon \sin\tau = T' \quad (21)$$

The graphical construction of T' goes as follows:

We mark a point r/a on the y-axis (point A, Fig. 15). From point A we describe a circle with radius ϵ, which intersects the line: $y = 1$ (line BC) in D. We draw a line AE through A and D. We now draw a tangent to the cycloid $FG \| AE$. Then draw AF parallel and FH perpendicular to the x-axis. Then:

$$OH = x' = \tau - \epsilon \sin\tau = T'$$

The construction is very easy if we use squared paper and parallel rules.

If we write the equation:

$$T_1 = q^{3/2}.T_1'/2\pi$$

in logarithmic form, we get:

$$\log T_1 = \tfrac{3}{2} \log q - \log 2\pi + \log T_1' \quad .. \quad (22)$$

We can solve this equation by means of a special slide-rule (Fig. 16). First we transform the linear scale A of the cycloidal nomogram into a logarithmic scale T'. The slide has two logarithmic scales for q, which have a relative length of 1·5 times scale T'. The fixed scale T, which corresponds with the undermost scale of the slide, is shifted to the right over a length of log 2π with regard to scale T'.

Example.

Assume that a body is following an elliptical orbit around the Sun, the elements of the ellipse being $a = 0{\cdot}625$ A.U., $\epsilon = 0{\cdot}645$. It is required to construct the period of time, in years, needed for the body to proceed from a point of radius $r_1 = 1{\cdot}0$ to the perihelion.

We have $r_1/a = 1{\cdot}0/0{\cdot}625 = 1{\cdot}6$.

Using the cycloid (Fig. 16), and knowing r_1/a and ϵ, we construct T' on scale A as described above, and find the value $T' = 2{\cdot}52$. We now set the slide in such a manner that the mark 1·0 of the uppermost scale of the slide coincides with the value 2·52 on scale T' (as shown).

Since $q = a = 0{\cdot}625$, we now determine the point on the lowest scale T coinciding with a point 0·625 on scale q. This point is 0·199, which is the time in years it takes the body to reach perihelion.

In this manner it is possible to determine the time T very rapidly, and, if the cycloid is drawn on a sufficiently large scale and the logarithmic scales are of sufficient length, an accuracy of about 0·3% is easily attained. The construction is less exact when the angle τ is nearly 0°, 90° or 180°. Note that for $r/a = 1$, $T' = \pi/2 - \epsilon$.

VI. PLOTTING AN ORBIT

A common task in astronautics will be to plot an orbit to a certain planet, starting from Earth at a certain moment. Let us assume that the radius of the planet's orbit is n A.U., and that the angle which the Earth and the planet are making at the moment of departure is ψ (Fig. 13). If T is the time in which the spaceship must reach the planet, then:

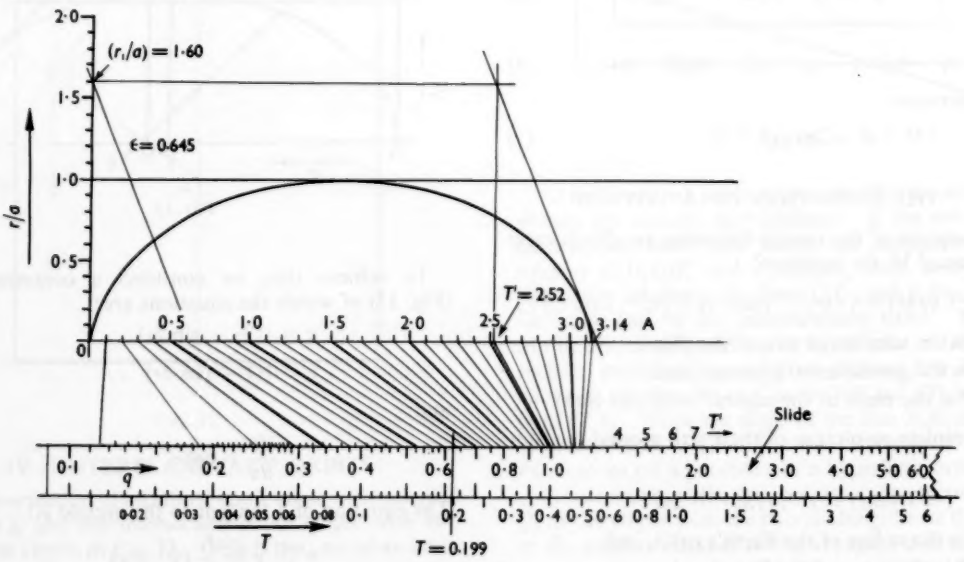

FIG. 16.

$$T = (q^{3/2}/2\pi).[(\tau_1 - \epsilon \sin \tau_1) - (\tau_2 - \sin \tau_2)]$$
$$= (q^{3/2}/2\pi).(T_1' - T_2')$$

Now the time which the spaceship, travelling from Q_1 to Q_2, takes to reach the planet, must, of course, be equal to the time which the planet takes to travel from Q_0 to Q_2.
Thus:

$$T = (n^{3/2}/2\pi)(\phi + \psi) \quad .. \quad .. \quad (23)$$

where ϕ is the angular distance of the voyage.
Then:

$$\phi = 2\pi T.n^{-3/2} - \psi \quad .. \quad .. \quad (24)$$

The value of the form $2\pi T.n^{-3/2}$ can be found by the use of the logarithmic scales.

To find an orbit which satisfies this condition we try several pairs of radii r_1 and r_2 for which the angular distance is ϕ. To achieve this we assume r_2 to make an angle ϕ_2 (Fig. 13) with the major axis, then:

$$\phi_1 = \phi_2 + \phi.$$

We now construct the points R_2 and R_1 and trace the slope-line CD. It is not necessary to trace the other lines. From this line we measure q and ϵ:

$$q = (DD' + CC')/2 \quad .. \quad .. \quad (25)$$

$$\epsilon = (DD' - CC')/(DD' + CC') \quad .. \quad (26)$$

From this we derive the time T, using the nomogram. We do this for three or four values of ϕ_2 and set off the resulting T's against the ϕ's in a graph.

We can determine which value of ϕ_2 tallies with the required value of T. Then for this value of ϕ_2 we can construct the slope-line and find the values of q and ϵ.

VII. THE DIRECTION AND THE MAGNITUDE OF THE IMPULSE

Now, as the elements of the ellipse are determined, we must find the direction and the magnitude of the impulse, which must be given to the spaceship for it to follow the prescribed orbit. These also can be constructed easily.

If we call the elliptical velocity of the spaceship at the moment of departure V_1 and the circular velocity of the Earth V_a, then:

$$V_1^2 = \mu(2/r_1 - 1/a) = V_a^2(2 - r_1/a)$$

where $\mu = fM$.

Thus:

$$V_1/V_a = (2 - r_1/a)^{\frac{1}{2}} \quad .. \quad .. \quad (27)$$

Now, in Fig. 17, the focal point of the ellipse is F, the radius of the orbit at the moment of departure is $FQ = r_1$, and WQ is the tangent at the ellipse in point Q. WF is the direction of the major axis of the ellipse.

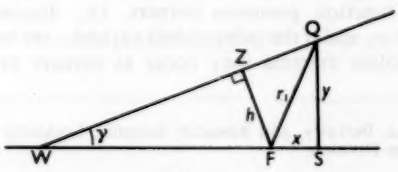

FIG. 17.

Then, if h is the length of the perpendicular from F on the tangent,

$$h^2 = WF^2 - WZ^2 = [(y/\tan\gamma) - x]^2 - h^2/\tan^2\gamma.$$

From which:

$$h = (y/\tan\gamma - x)/(1 + 1/\tan^2\gamma)^{\frac{1}{2}} \quad .. \quad .. \quad (28)$$

Now according to Equation (4):

$$\tan\gamma = (r_1\epsilon - x)/y, \text{ and } y = (r_1^2 - x^2)^{\frac{1}{2}},$$

and, derived from Equation (2):

$$x = (r_1 - p)/\epsilon.$$

By substituting these relations in Equation (28) we find after some transformation:

$$h^2 = r_1 p/[2 - r_1(1 - \epsilon^2)/p] \quad .. \quad (29)$$

And, as:

$$p = a(1 - \epsilon^2),$$

we find:

$$h^2 = r_1 p/(2 - r_1/a) \quad .. \quad .. \quad (30)$$

Combining Equations (27) and (30), we find:

$$V_1/V_a = (\sqrt{r_1 p})/h$$

Starting from this equation, V_1 can be easily constructed, Fig. 18.

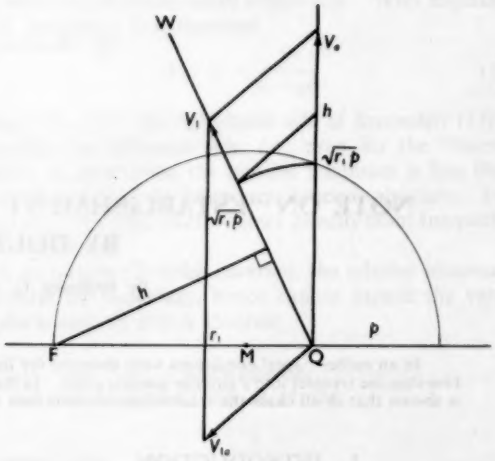

FIG. 18.

We prolong $r_1 = FQ$ with a length p (FH in Fig. 13), bisect $(r_1 + p)$ and draw a circle with radius $(r_2 + p)/2$ from the centre M. In Q we erect a perpendicular on FQ; the length of this line up to the point of intersection with the circle is $\sqrt{r_1 p}$.

Now we choose a length on the perpendicular which represents the velocity V_a. If we make $V_a = 29.8$ cm., then all velocities have the scale of 1 cm. per km./sec. The direction of the velocity V_1 is the tangent QW to the

ellipse in point Q. By constructing a parallelogram of velocities we find the direction of the velocity V_{1a} and the magnitude of the impulse, expressed as a velocity, which we must give the spaceship in order to adapt its velocity to the velocity on the elliptical orbit on the point of departure.

At the point of arrival our spaceship must adapt its velocity to the velocity of the planet of destination. The circular velocity of this planet is:

$$V_p^2 = \mu/r_2 = \mu/(n.r_1) = V_a^2/n,$$

or:

$$V_p/V_a = 1/\sqrt{n}.$$

This value can be constructed as shown in Fig. 19, which needs no elucidation.

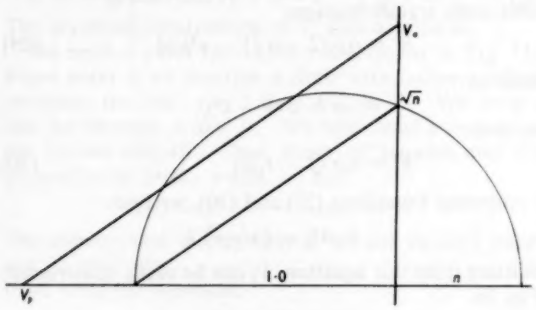

FIG. 19.

Then the velocity V_{2p}, necessary to adapt the velocity of the spaceship to the velocity of the planet can be contructed in the same manner as the velocity V_{1a}.

LIST OF SYMBOLS

a	semi major axis of a conic section.
b	semi minor axis of a conic section.
f	gravitational constant.
h	perpendicular from the focus on a tangent.
n	radius of the orbit of a planet (A.U.).
p	parameter of a conic section.
q	semi major axis of a conic section (A.U.).
r	radius of a conic section.
x, y	rectangular coordinates.
M	mass of central body (the Sun).
T	duration of voyage along elliptic orbit.
V	velocity of spaceship.
V_a	circular velocity of Earth.
V_p	circular velocity of planet of destination.
V_{1a}, V_{2p}	transfer velocities.
γ	angle between tangent and major axis.
ϵ	eccentricity of a conic section.
$\mu = fM$.	
τ = arc cos$[(1 - r/a)/\epsilon]$.	
ϕ	angular distance to pericentre of a point on a conic section.
ϕ	angular distance of a voyage.
ψ	angular distance between Earth and planet at moment of departure.

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1. M. Vertregt, *J.B.I.S.*, 1957–58, **16**, 326.

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NOTE ON "ESTABLISHMENT OF A CIRCULAR SATELLITE ORBIT BY DOUBLE IMPULSE"*

By Professor G. LEITMANN,† Ph.D., Fellow

ABSTRACT

In an earlier paper[1] conditions were deduced for the existence of a relative minimum value of propellent expended in a two-impulse transfer into a circular satellite orbit. In this note the relation of relative and absolute minima is investigated. It is shown that in all cases the relative minimum is also the absolute one.

I. INTRODUCTION

AN earlier paper[1] dealt with the determination of the direction of launch and the staging ratio leading to minimum propellent consumption for a two-stage, impulsive rocket required to carry a given payload into a circular orbit of prescribed radius. Launch from rest and from the rotating Earth were discussed. Conditions yielding a stationary (relative) minimum value of propellent consumption were determined and found to depend critically on the ratio of the specific impulses of the two stages.

If a function possesses corners, *i.e.*, discontinuous derivatives, and if the independent variables are bounded, the absolute extrema may occur at corners or at the

* Manuscript received 22 August, 1959.

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boundary of the domain. Hence, it is not clear *a priori* that the relative minimum found in reference (1) is the absolute minimum. It is the purpose of this note to investigate whether or not the absolute minimum value of fuel consumed is indeed the relative minimum value previously deduced. The nomenclature of this note is the same as that of reference (1).

II. COMPARISON OF MINIMUM VALUES

The function to be minimized is:

$$F = \frac{1}{c_1}\sqrt{V_x^2 + V_y^2 + V_z^2} - \frac{1}{c_2}\frac{r_1}{r_2}\sqrt{p(A - V_z^2)} + \frac{1}{c_2}\sqrt{r_2 g_2} \quad .. \quad (1)$$

where

$$V_x^2 + (V_y + v_E)^2 = p\,(A - V_z^2) \quad .. \quad (2)$$

For real values of velocity the arguments of the square roots in Equation (1) cannot pass through zero in the interior of the domain. Hence F possesses no corners and we need investigate only its values at the boundary of the domain. From Equation (2) it is seen that the independent variables chosen, V_x and V_z, are bounded by

$$V_x^2 + pV_z^2 = pA \quad .. \quad .. \quad (3)$$

so that

$$0 \leq V_x^2 \leq pA, \quad 0 \leq V_z^2 \leq A \quad .. \quad (4)$$

Since F possesses only one extremum with respect to V_x, at $V_x = 0$ —and it was shown to be a minimum— we need concern ourselves with the values of F only at

$$V_x = 0 \text{ and } V_z = 0,\ V_z = \pm\sqrt{A} \quad .. \quad (5)$$

Of these values of F the one at $V_x = V_z = 0$ is an extremum so that only the one at $V_x = 0$, $V_z = \sqrt{A}$ (the negative value of V_z is of no interest in physically realistic problems) must be found. It is

$$F = \frac{1}{c_1}\sqrt{v_E^2 + A} + \frac{1}{c_2}\sqrt{r_2 g_2} \quad .. \quad (6)$$

F possesses two extrema and for

$$\frac{c_1}{c_2}\frac{r_1}{r_2} \geq \left(\frac{r_1}{r_2}\right)^2 - \frac{v_E}{p(\sqrt{pA} - v_E)} \quad .. \quad (7)$$

the minimum occurs at

$$V_x = V_z = 0 \quad .. \quad .. \quad (8)$$

and is given by

$$F = \frac{1}{c_1}\left[\sqrt{pA}\left(1 - \frac{c_1}{c_2}\frac{r_1}{r_2}\right) - v_E\right] + \frac{1}{c_2}\sqrt{r_2 g_2} \quad (9)$$

In comparing the values of F given by Equations (6) and (9), respectively, we ask if

$$\sqrt{v_E^2 + A} \gtrless \sqrt{pA}\left(1 - \frac{c_1}{c_2}\frac{r_1}{r_2}\right) - v_E \quad .. \quad (10)$$

To answer this question we must choose values for c_1/c_2 and v_E which make the right-hand side of Inequality (10) largest. With respect to c_1/c_2 this means choice of the equality sign in Inequality (7). In that case both sides of Inequality (10) are positive so that we may square them. Thus, we have in place of Inequality (10):

$$1 \gtrless (1 - \rho^2)(1 - x)^{-2} - \frac{2x}{1 - x} \quad .. \quad (11)$$

where

$$\rho = \frac{r_1}{r_2}, \quad x = \frac{v_E}{v_e}\sqrt{1 + \rho}$$

For any ρ, the right-hand side of Inequality (11) takes on its largest value when

$$x = \rho^2 \quad .. \quad .. \quad .. \quad (12)$$

We may note that this corresponds to actual values of v_E only for sufficiently small values of ρ. With Equation (12), Inequality (11) becomes

$$1 \gtrless 1 - \frac{\rho^2}{1 - \rho^2} \quad .. \quad .. \quad (13)$$

Thus, for $\rho < 1$ the right-hand side of Inequality (13) is less than its left-hand side, *i.e.*, even for the "worst" choice of parameters the relative minimum is less than the value of F at the boundary, hence is absolute. For start from rest this result follows directly from Inequality (10), of course.

If Inequality (7) holds reversed, the relative minimum precedes the boundary, hence cannot exceed the value at the boundary and is absolute.

REFERENCE

1. G. Leitmann, *J.B.I.S.*, 1959–60, **17**, 194.

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A NOTE ON RELAY SATELLITES*

By S. W. GREENWOOD,† B.Sc., M.Eng., A.M.I.Mech.E., A.F.R.Ae.S., Member of Council

ABSTRACT

Clarke's proposal[1] for the establishment of "stationary" relay satellites in the 24-hr. circular equatorial orbit is extended to the other planets of the solar system. General expressions for the orbit radius and the area of surface visible from the satellite are derived and values for each planet tabulated.

INTEREST has long been expressed in the 24-hr. circular orbit around the Earth in its equatorial plane, partly because of the novelty of the prospect of a free body rotating around the Earth at the same angular velocity as that of the Earth about its own axis, and partly because of the practical possibilities of using such a satellite as a relay station for radio transmission over the portion of the Earth's surface visible from it.[1]

In contemplating the extension of this principle to other planets in the solar system, it is interesting to consider whether any new aspects of the problem arise. The dynamics of the artificial satellite's motion would clearly be the same—it would be necessary to move in the equatorial plane of the planet at the same angular velocity as that of the planet about its axis, and hence in a circular path about the planet. A significant difference in viewpoint, however, is that the setting up of the satellites will almost certainly have preceded landings on the planet concerned. It seems to the author that the following specific advantages would be associated with the selection of the type of orbit under consideration:

(*a*) Continuous observation of a selected area of the planet over a prolonged period would be possible. Such an area might be of interest either for scientific reasons, or because it was being considered as a possible area for landing and preliminary exploration.

(*b*) In the period following a landing, the satellite could maintain continuous contact with the exploring party visually and by radio, and could act as a relay station for contact between ground parties widely separated from one another.

These suggestions are of a general character, and will obviously be dependent upon sufficiently favourable circumstances in any given case for their being put into practice.

Data for the planets of the solar system are presented in Table I. Caution should be exercised in connection

TABLE I

Body	t, hr.	R_0, miles	a	R, miles	$\frac{R}{R_0}$	$\frac{A}{A_{max}}$	Notes (see below)
Mercury	88 × 24	1550	0·36	197,000	127	0·99	
Venus	13 × 24	3850	0·87	136,000	35·3	0·97	1
Earth	24	3950	1	26,200	6·63	0·85	
Mars	24·62	2100	0·38	12,660	6·03	0·83	
Jupiter	9·84	44,350	2·64	100,500	2·27	0·56	2
Saturn	10·23	37,550	1·13	69,300	1·85	0·46	2
Uranus	10·82	14,650	1·07	37,800	2·58	0·61	2
Neptune	14	13,850	1·41	47,400	3·42	0·71	2
Pluto	6·4 × 24	2000	0·5	45,600	22·8	0·96	3

LIST OF SYMBOLS

t = time for one rotation in orbit, hours.
R_0 = radius of planet, miles.
a = $g_0/(g_0)_{Earth}$
g_0 = gravitational acceleration at the planet's surface.
R = radius of orbit, miles.
A = area of surface visible from satellite.
A_{max} = area of one hemisphere of the planet's surface.

NOTES:

1. Rotational period very uncertain.
2. The "Giants" do not rotate as solid bodies. The rotational period is given for the equatorial region.
3. All data very uncertain. Surface gravitational acceleration estimated assuming planet has same mean density as the Earth.

* Manuscript received 24 December, 1958.

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with much of the material included in the table, owing to the unreliability of some planetary data. The figures are, therefore, approximate and will have to be revised and the derived data calculated more accurately at a later date.

The radius of the orbit under consideration is given by:

$$R^{3/2} = 44{\cdot}7\ tR_0 a^{1/2} \quad .. \quad .. \quad (1)$$

The fraction of the area of a hemisphere of the planet's surface that may be seen from the satellite is given by:

$$A/A_{\max} = 1 - R_0/R \quad .. \quad .. \quad (2)$$

These relationships are derived in the Appendix.

The results of the calculations also are presented in Table I. The planets appear to form three distinct groups for the purposes of this study.

Group 1: Mercury, Venus and Pluto.

The relay satellite orbits are at large distances from these planets, mainly owing to their long rotational periods. The effect of this is to permit almost a complete hemisphere to be viewed in each case, but at such a large distance that the value of doing it is questionable.

Group 2: Earth and Mars.

The orbits of relay satellites for these planets are relatively closer, and a relatively large proportion, over 80%, of a hemisphere of the planet is visible. The use of such satellites appears quite practicable here. Clarke[1] has proposed three spaced at 120° to each other for substantially complete radio coverage of the Earth.

Group 3: Jupiter, Saturn, Uranus and Neptune.

All the "Giants" rotate rapidly, and the associated relay satellite orbits lie fairly close to the planets. This makes the satellites excellent observation stations, but reduces the extent of the area visible to about half a hemisphere. The orbits lie within the orbits of the closest known moons of the planets, but it may be that small moons do in fact exist in the regions of interest which could be used as bases. In the case of Saturn, the orbit lies within the ring system and it is debatable whether such an arrangement would be practicable.

If a leisurely and more extensive visual inspection of the planet is contemplated in place of a prolonged examination of a particular area, this might be accomplished by moving in an orbit at a slightly different radius from the one considered here. By a suitable selection of orbit, the planet will appear to the observer to rotate at any desired rate within limits.

Studies similar to the above were carried out for some of the larger moons in the solar system, but without success. The influence of the gravitational field of the planet concerned would make the establishment of a circular orbit around the moon at the distances involved impossible without the application of power.

ACKNOWLEDGMENT

The author is grateful to Mr. Patrick Moore, F.R.A.S., for assistance in supplying data on the planets and for his helpful comments.

APPENDIX

1. *Assumption*

The orbit of the satellite is only affected by the gravitational field of the planet around which it rotates and is circular.

2. *Derivation of Equation* 1

Putting t = time for one revolution in orbit
R = radius of orbit
R_0 = radius of planet
g_0 = gravitational acceleration at the planet's surface

Then $$t = \frac{2\pi\ R^{3/2}}{R_0\, g_0^{1/2}}$$

Rearranging, $$R^{3/2} = \frac{t\ R_0\, g_0^{1/2}}{2\pi}$$

For purposes of computation, with R and R_0 measured in miles and t in hr., this may be written

$$R^{3/2} = 44{\cdot}7\ tR_0 a^{1/2}$$

where $a = g_0/(g_0)_{\text{Earth}}$

3. *Derivation of Equation* 2

Area of surface visible from satellite,

$$A = \int_0^{\beta} 2\pi\ R_0^2 \sin\alpha\ d\alpha$$

Where β is the complement of the angle between lines drawn from the satellite (a) to the centre of the planet and (b) tangential to the surface of the planet respectively.

The integral becomes

$$A = 2\pi\ R_0^2\ (1 - R_0/R)$$

The maximum possible area visible is the area of a hemisphere,

$$A_{\max} = 2\pi\ R_0^2$$

Thus $$A/A_{\max} = 1 - R_0/R$$

REFERENCES

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OPTIMUM FREQUENCY SELECTION FOR INTERPLANETARY COMMUNICATION*

By Professor SURESH CHANDRA,† M.Sc.

ABSTRACT

The power required for one-way radio communication from Earth to various planetary bodies has been calculated at various frequencies by choosing suitable minimum detectable power, molecular atmospheric absorption, receiver bandwidth and antenna gains. It has been found that Jupiter is not easily accessible to radio communication, owing to its large distance and heavy absorption by ammonia present in its atmosphere. However, as discussed in the paper, short-interval contacts may be established at about 100 Mc./sec. under most favourable conditions. A frequency of about 3000 Mc./sec. has been recommended for communication with the Moon, Mars and Venus. Under the conditions assumed in this paper the transmitter power required for communication with the Moon, Mars, Venus and Jupiter at 3000 Mc./sec. using conventional receivers is 7 W., 230 kW., 70 kW. and 8·5 MW. respectively. The power requirement for the transmitter decreases if a receiver employing a maser preamplifier is used. The power of the transmitter at 3000 Mc./sec. calculated for such receivers with a noise temperature of 85° K. has been obtained as 0·37 W., 12·3 kW., 3·7 kW and 453 kW. for the Moon, Mars, Venus and Jupiter respectively.

I. INTRODUCTION

THE first successful attempt in the direction of interplanetary radio communication was reported by Mofenson[1]. He used a radar set of 3 kW. peak power, transmitting ¼-sec. pulse at 111·5 Mc./sec. and received the echoes from the Moon. Similar results have also been reported by De Witt and Stodola[2]. Clarke[3] discussed the possibilities of Earth-to-spaceship, planet-to-planet, and spaceship-to-spaceship radio communication. Kerr[4] made a theoretical speculation on the possibility of getting echoes from the Sun and other planets at 30 Mc./sec. He has indicated the possible use of higher frequencies in the case of planets. The work reported here has been done with a view to finding suitable frequencies for such communication between different planets. Our main interest lies in using planets as one-way radio communication circuits or as relaying reflectors rather than getting echoes.

II. GENERAL CONSIDERATIONS

A signal can only be detected when the power reaching the receiver is such that its ratio to the Johnson noise is at least equal to the receiver noise figure. The received signal power at a particular point will depend upon its distance from the transmitter, the transmitting and receiving antenna gains, and the molecular absorption of electromagnetic energy by gases in the path.

Total absorption of radio waves (microwaves) in various planetary atmospheres has been reported elsewhere[5]. Power reaching a particular point can be calculated in terms of receiving and transmitting antenna gains in the following manner.

The density of the power at a distance of r metres due to a transmitter which delivers a peak power of P_t watts to an isotropic radiator is given by the total power divided by the surface area of the sphere of radius r.

However, if the transmitter is fitted with a directive antenna with maximum power gain G_t, then the power density P_d in the direction of maximum gain is:

$$P_d = \frac{G_t.P_t}{4\pi r^2} \text{ W./m.}^2 \quad .. \quad .. \quad (1)$$

This power density is gathered by the effective absorption cross-section of the receiving antenna, A m.², and generates at the receiver input a power of magnitude

$$P_r = A.P_d \text{ W.} \quad .. \quad .. \quad (2)$$

The absorption cross-section is related to the maximum receiver antenna gain G_r by the following:

$$A = \frac{G_r.\lambda^2}{4\pi} \text{ m.}^2 \quad .. \quad .. \quad (3)$$

where λ is the wavelength of the wave radiated in metres.

From (1), (2) and (3) we get the power at the receiver input as

$$P_r = \frac{G_r.G_t.P_t.\lambda^2}{(4\pi r)^2} \text{ W.} \quad .. \quad .. \quad (4)$$

The power of the transmitter which produces a power P_r at the receiver is given by the expression:

$$P_t = \frac{P_r.(4\pi r)^2}{G_r.G_t.\lambda^2} \text{ W.} \quad .. \quad .. \quad (5)$$

Now if the molecular absorption in the path is α db., then the power of the transmitter P'_t required to produce the same power P_r at the receiver is

$$P'_t = P_t.\text{Antilog}\,(\alpha/10) = \frac{P_r.(4\pi r)^2}{G_r.G_t.\lambda^2}\,\text{Antilog}\,(\alpha/10) \text{ W.} \quad (6)$$

By substituting the suitable values of G_r, G_t and minimum detectable power P_r, we can have an idea of the transmitter power required for communication between Earth and various heavenly bodies.

A different method of calculating the power of the transmitter is from the formula for the communication capacity of a channel[6]

$$C = W.\log_2\left(1 + \frac{\overline{S_r}}{N'_0 W}\right) \quad .. \quad (7)$$

* Manuscript received 15 December, 1958, and in revised form 12 November, 1959.

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where $\overline{S_r}$ = average received signals,
W = channel bandwidth,
N_0' = noise power density,
C = maximum or ideal rate of transmission in bits/sec. (if W is in c./sec.).

Another useful relation which can be derived from (7) is:

$$C < \frac{1{\cdot}4\,\overline{S_r}}{N_0} \qquad .. \qquad .. \qquad (8)$$

Swerling[6] has used Equation (8) to estimate the power requirements for space communication purposes.

III. CHOICE OF P_r

The minimum signal power which can be detected by a receiver can readily be calculated from the formula for noise figure:

$$NF = \frac{S/kTB}{S_0/N_0} \qquad .. \qquad .. \qquad (9)$$

where NF = noise figure of the receiver,
S = available signal power from signal source,
S_0 = available signal power from receiver,
N_0 = available noise power from receiver,
T = Absolute temperature,
B = Noise bandwidth of the receiver,
k = Boltzmann constant.

The minimum signal power which can be received corresponds to the minimum value of S_0/N_0, which is unity. Substituting $S_0/N_0 = 1$ in (9), the expression for NF reduces to:

$$NF = \frac{(P_r)_{\text{min.}}}{kTB}$$

If the noise figure is given in decibels,

$$(P_r)_{\text{min.}} = kTB.\ \text{Antilog}\,\frac{(NF)}{10} \qquad .. \qquad (10)$$

An average value of 15 db. has been taken for the receiver noise figure in calculations made here.

Recently Townes *et al.*[7] have described the design and operating characteristics of a maser radiometer for use in radio astronomy at X band frequencies. The operating system described has a bandwidth of 5·5 Mc./sec. and input noise temperature, including background radiation into the antenna, of about 85° K. The limitation on noise in such a system lies in the thermal radiation from components preceding the amplifier, in particular from the ferrite switch, the circulator and the hot ground seen by the antenna. A careful design may largely eliminate these noises, so that a noise temperature near 10° K. should be obtained. A noise temperature of 85° K. has been taken in our calculations. The noise figure is related with the input noise temperature by the following relation

$$\text{Input noise temperature} = (NF-2) \times 290°\ \text{K}. \qquad (11)$$

In a practical case it is advisable to choose the value of $P_r = 10 \times (P_r)_{\text{min.}}$ This is done to take into account the following factors:

(1) Temperature used for the calculation of the Johnson noise is the average temperature ($T_{\text{max.}}/\sqrt{2}$) of the planet and not the maximum temperature recorded. Hence the instantaneous noise will be greater than the theoretical average value.

(2) Noise level will increase when the planets come in line with a radiating body, such as the Sun.

(3) Fluctuations and disturbances in the atmosphere due to sudden galactic bursts will cause reduction in signal intensity.

It may be pointed out here that such a signal strength will be well above the total noise level obtained as a combined effect of Johnson, galactic and solar noises.

IV. CHOICE OF ANTENNA GAIN

Since the main interest is interplanetary communication, it is important that the antenna structure be of such a type that long-period contacts are established. The situation allows only two types of structures, *viz.* (1) isotropic radiators, and (2) directive radiators which can be easily rotated to maintain the contact once established. The antenna gain should be as high as possible consistent with the limitation imposed by the second requirement.

At 100, 500, and 1000 Mc./sec., high-gain arrays will be too big in structure and at a particular setting the transmitter will be in contact with the planet for a limited interval of time. The structure will have to be rotated, in order to maintain the contact with the planet. This is very inconvenient when the structures are very big. At these frequencies a simple isotropic transmitter antenna has been chosen (antenna gain 1). This has the disadvantage that a large proportion of the power will be wasted. The receiving antenna gains have been taken as 250, 500 and 1000 at 100, 500 and 1000 Mc./sec. respectively, since these can be realized in practice.

In the case of 3000, 10,000 and 30,000 Mc./sec., the antenna structures will be more compact and rotation for continuous contact is no longer a problem. At these frequencies the antenna gain can reasonably be taken as 2000. Owing to their high directivity these antennae avoid stray disturbances caused by delayed signals arriving as echoes from other planets.

V. CHOICE OF BANDWIDTH

Any value of the bandwidth can be taken depending upon the purpose for which the communication is made. The narrower the bandwidth, the smaller will be the power requirements for the transmitter (Equation (7)). In the present calculations a tentative value of 10 kc./sec. has been taken; this is a good approximation for speech-modulated transmission.

The bandwidth of 10 kc./sec. is required for speech transmission. It is most unlikely that this type of communication will be of a personal nature, making voice-to-voice contact a necessity. The teletype system of transmission is quite suited for the transference of technical data[8]. Above all, common experience of radar transmission shows that the signalling factor with pulsed power depends upon the peak power delivered and not the average power. This simplifies the problem of getting high average powers.

In a very narrow bandwidth receiver, the setting of the receiver should take into account the Doppler shift. The Doppler frequency change is approximately given by

$$\Delta f = f.V_0/V$$

where V_0 is the relative velocity of the target and V is the velocity of the electromagnetic wave.

Assuming the relative velocity of the Moon with respect to the Earth, due to the rotation of the Earth and the motion of the Moon in its orbit, to be 1000 miles/hr. an idea of the Doppler shift can be had from Table I.

TABLE I.—*Doppler Frequency Shift for Different Frequencies (Earth-Moon Communication)*

Frequency, f, Mc./sec.	Change in frequency, Δf, in kc./sec.
100	0·15
500	0·75
1000	1·50
3000	4·50
10,000	15·0
30,000	45·0

VI. ABSORPTION IN THE PLANETARY ATMOSPHERES

The data for the molecular absorption have been taken from an earlier paper by Chandra and Srivastava[5] on this subject. Table II shows the absorption in the planetary atmospheres of different planets at various frequencies.

TABLE II.—*Total Absorption of Radio Waves in Planetary Atmospheres*

Frequency, Mc./sec.	Absorption, db.				
	Earth	Moon	Mars	Venus	Jupiter
100	0·26	—	—	—	—
500	0·01	—	—	—	—
1000	0·003	—	—	—	0·376
3000	0·092	—	0·002	0·003	0·505
10,000	0·099	—	0·019	0·037	6·374
30,000	0·160	—	0·087	0·170	29·498

VII. RESULTS

The data presented above have been utilised for computing the minimum power of a transmitter on Earth whose signals can be detected on various heavenly bodies. The results for an ordinary receiver are given in Table III. The results for a radiometer using masers are given in Table IV.

TABLE III.—*Power Required for One-Way Interplanetary Communication Using Conventional Receivers*

Frequency, Mc./sec.	Minimum power for the Earth's transmitter (kW.) for one-way communication to			
	Moon	Mars	Venus	Jupiter
100	$1·3 \times 10^{-1}$	$4·3 \times 10^{3}$	$1·25 \times 10^{3}$	$1·35 \times 10^{5}$
500	1·53	$5·0 \times 10^{3}$	$1·45 \times 10^{3}$	$1·6 \times 10^{4}$
1000	3·04	$1·0 \times 10^{4}$	$2·95 \times 10^{3}$	$3·5 \times 10^{4}$
3000	$7·0 \times 10^{-3}$	$2·3 \times 10^{2}$	$7·0 \times 10$	$8·5 \times 10^{3}$
10,000	$8·0 \times 10^{-2}$	$2·6 \times 10^{3}$	$7·5 \times 10^{2}$	$3·6 \times 10^{3}$
30,000	$7·2 \times 10^{-1}$	$2·5 \times 10^{4}$	$7·5 \times 10^{3}$	$7·0 \times 10^{4}$

TABLE IV.—*Power Required for One-Way Radio Communication Using Receiver with Maser Preamplifier*

Frequency, Mc./sec.	Minimum power required for the Earth's transmitter (kW.) for one-way radio communication to			
	Moon	Mars	Venus	Jupiter
100*	—	—	—	—
500	$8·13 \times 10^{-2}$	$2·67 \times 10^{2}$	$7·73 \times 10$	$8·53 \times 10^{4}$
1000	$1·62 \times 10^{-1}$	$5·33 \times 10^{2}$	$1·57 \times 10^{2}$	$1·87 \times 10^{5}$
3000	$3·7 \times 10^{-4}$	$1·23 \times 10$	3·73	$4·53 \times 10^{2}$
10,000	$4·26 \times 10^{-3}$	$1·39 \times 10^{2}$	$3·99 \times 10$	$1·88 \times 10^{7}$
30,000*	—	—	—	—

* No maser operating at these frequencies has been announced.

VIII. CONCLUSIONS

The results presented in Table III indicate that radio communication with Jupiter under the conditions assumed in this paper requires a very large power which seems practically impossible. The higher microwave frequencies are attenuated owing to the presence of ammonia in its atmosphere. The power requirement may come within practical approach at a frequency of 3000 Mc./sec. if the bandwidth of the receiver is further reduced. The power requirement for 100 Mc./sec. may also be brought within practical approach if a directive transmitting antenna similar to that at the receiver is used instead of the isotropic type, and if the bandwidth is further reduced. This will involve a very heavy structure which cannot be conveniently rotated for continuous contact. Thus communication with Jupiter at 100 Mc./sec., if made possible, will be only for short intervals.

The power requirement of a transmitter for communication with Mars and Venus is least at 3000 Mc./sec., as compared to the power at other frequencies. Hence it is concluded that a frequency near 3000 Mc./sec., will be the most suitable for radio communication with these two planets. However, it may be mentioned that the power requirement at 100 Mc./sec. can be less than the

value at 3000 Mc./sec. provided an antenna with an approximate gain of twenty or more is used. Such an antenna will also have a fairly big structure and cannot be so conveniently rotated as a paraboloid at 3000 Mc./sec. Further, the beam width of a 100-Mc./sec. antenna will be much larger than the angular diameters of the planets and thus some of the drawbacks of the isotropic antenna (disturbances due to delayed signals, etc.) will arise.

For the Moon the power requirements are not very heavy at any of the frequencies and therefore any frequency may be used for communication. However, the use of 3000 Mc./sec. is recommended for the reasons already discussed above.

Using a maser preamplifier, the power requirements will be much less because it can detect weak signals in noise. A comparison can be had from Tables III and IV. The calculations for 100 and 30,000 Mc./sec. have not been done because no masers operating at these frequencies have been announced.

In all these discussions frequencies below 100 Mc./sec. have not been considered because at these frequencies the reflection and refraction effects in the ionosphere cannot be overlooked and it is probable that the signals may not reach other planets, as pointed out by De Witt and Stodola[2].

NOTE ADDED IN PROOF

Since the acceptance of the above paper a large volume of information has been published in the April, 1960, issue of *Proceedings of the Institute of Radio Engineers* on Space Electronics. It has, therefore, become necessary to give a brief comment on the validity of my conclusions in the light of the information contained there. It may be stated that all the papers on this subject in this issue indicate that the best frequency region for space communication lies between 1000 and 10,000 Mc./sec. and the frequency suggested in the present paper is 3000 Mc./sec. The following summary of the I.R.E. papers will be of interest in connection with the present conclusions:

Mueller,[9] on the basis of an empirical relationship connecting mass, power, gain, and frequency of the parabolic antenna and transmitting system for space-vehicle-to-Earth communication, has concluded that the higher the frequency the greater will be the effective transmitted power. He takes the lowest possible frequency as 400 Mc./sec. and sets up an upper limit at 10,000 Mc./sec., owing to increased problems associated with attitude control as the antenna bandwidth narrows and the increased mechanical tolerance at high frequencies.

The spectral characteristics of various galactic and planetary radio noise sources are discussed by Smith,[10] and it is suggested that the future of long-range communication lies in the microwave region.

Ryerson[11] favours the use of passive satellite communication in comparison with the active system. The operating spectrum is assumed to lie between 2000 and 10,000 Mc./sec. The lower limit is chosen on the basis of the antenna gain and size as well as the complication arising from possible diffraction from the passive system and the ionospheric effects. The upper limit is set by the accuracy of fabrication necessary to avoid phase cancellation and multipath effects as well as atmospheric attenuation considerations.

The possibility of world-wide use of the "distributed" passive relay system is discussed by Bond *et al.*[12] in their paper "Interference and Channel Allocation Problems Associated with Orbiting Satellite Relays." The feasible operating spectrum is assumed to be 2000–10,000 Mc./sec.

Dimond[13] takes 1000–10,000 Mc./sec. as the available transmission "window," on noise and atmospheric absorption considerations. The state of the art in present receiver and transmitter design has led to S band telemetry systems being favoured. A 2250-Mc./sec. telemetry system which is being attempted is considered.

Brockman *et al.*[14] describe the TRAC(E) system for *Pioneer III* and *IV*, operating at 960·5 Mc./sec., chosen on the basis of solar and galactic noise, atmospheric absorption and the present state of the art. Calculation of the absorption of microwaves due to water vapour in the Earth's atmosphere, showed that it was negligible at the recommended frequency of 3000 Mc./sec. Its contribution was approximately 5% of the total at 10,000 Mc./sec. and 30% at 30,000 Mc./sec. This is a further indication of the advantage of using a frequency of about 3000 Mc./sec.

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LIST OF SYMBOLS

A	effective absorption cross-section of receiving antenna, m².
B	noise bandwidth of receiver, Mc./sec.
C	maximum rate of transmission, bits/sec.
G_r	maximum receiver antenna gain.
G_t	maximum transmitting antenna gain.
N_0	available noise power from receiver, W.
N_0'	noise power density.
NF	noise figure, db.
P_d	power density, W./m.².
P_r	receiver input power, W.
$(P_r)_{\min.}$	minimum detectable input power, W.
P_t, P_t'	transmitter power, W.
S	available signal power from signal source, W.
S_0	available signal power from receiver, W.
S_r	average received signals.
T	absolute temperature, ° K.
$T_{\max.}$	maximum temperature recorded, ° K.
V	electromagnetic wave velocity, m./sec.
V_0	relative velocity of target planet, m./sec.
W	channel bandwidth, c./sec.
f	frequency, Mc./sec.
Δf	Doppler frequency change, kc./sec.
k	Boltzmann constant.
r	distance, m.
α	molecular absorption, db.
λ	wavelength, m.

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RELATIVE EFFECTS OF PROPELLENT DENSITY AND OF SPECIFIC IMPULSE ON ROCKET PERFORMANCE*

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(Communication from Aerojet-General Corporation)

ABSTRACT

The relative importance of propellent density and of propellent specific impulse is analysed for three different design situations. In each case a simple relationship for best performance is given in terms of the rocket component masses. The best mixture ratio of a liquid propellent (or formulation of a solid propellent) is determined.

I. INTRODUCTION

THE most important performance parameters of a propellent are its specific impulse I_{sp} and its density ρ. If one propellent is superior to another in both of these respects, it will certainly yield a better missile performance. However, if the density is greater and the specific impulse less, or *vice versa*, the effect on the performance is more problematical. It has become customary to select propellents for maximum $\rho^n I_{sp}$, where n may vary from 0 to about 1·5. The following analysis will show how the correct value of n is to be chosen.

Three design situations will be considered; in all of them the rocket will accelerate rapidly to a high burnout speed and will then coast ballistically or aerodynamically; cruise-type rockets are therefore excluded. In the first situation, part of the mass of the empty rocket is fixed and the propellent volume is fixed. In the second situation part of the mass of the empty rocket is fixed and the full mass is fixed. The third situation allows a varying diameter, but fixes the full mass, the length and part of the empty mass.

In the first two cases the intention is to maximize the burnout speed v of the rocket, which is given in terms of the propellent jet speed (j) and the masses of the rocket when full and when empty (m_f and m_e, respectively) to a first approximation by:

$$v = j \ln (m_f/m_e) \qquad .. \qquad .. \qquad (1)$$

The application in these two cases can be to a ballistic missile, or to an air-to-air boost-glide missile with fixed diameter; in both cases the range is an increasing function of boost speed, so it is sufficient to maximize the latter. The third case involves a boost-glide missile of variable diameter. Here also the intention is to maximize the glide range, which is roughly proportional to the product of the empty mass, some power of the boost speed, and the inverse square of the missile diameter.

II. PROPELLENT VOLUME AND PART OF EMPTY MASS FIXED

The first case we shall consider is a rocket with an empty mass composed of two parts. The first part, m_{e1}, is fixed; this might correspond to the payload and a portion of the propulsion system. The second part, corresponding to the propellent chamber, has a mass m_{e2} proportional to the propellent volume V. The volume V will be taken fixed, so that m_{e2} is fixed, and therefore also m_e. The propellent has density ρ, and mass $m_p = \rho V$. Therefore

$$v = j \ln \frac{m_f}{m_e} = j \ln \frac{m_e + \rho V}{m_e} \qquad .. \qquad (2)$$

If the propellent undergoes small changes $d\rho$ in density and dj in jet speed, the corresponding change in burnout speed dv is given by the relation

$$dv = \left[\ln \frac{m_e + \rho V}{m_e}\right] dj + j \frac{m_e}{m_e + \rho V} \cdot \frac{V}{m_e} d\rho \qquad (3)$$

If the effects of $d\rho$ and dj cancel, so that there is no change in v, we have

$$\frac{dj}{j} + \frac{\dfrac{m_e}{m_f} \cdot \dfrac{m_p}{m_e}}{\ln (m_f/m_e)} \cdot \frac{d\rho}{\rho} = 0 \qquad .. \qquad .. \qquad (4)$$

We must now find the value of n in

$$\rho^n I_{sp} = \text{constant} \qquad .. \qquad .. \qquad (5)$$

which gives the same result. We rewrite (5)

$$\rho^n j = \text{constant}$$

and differentiate the logarithm of this expression, obtaining

$$\frac{dj}{j} + n \frac{d\rho}{\rho} = 0 \qquad .. \qquad .. \qquad (6)$$

Comparing (4) and (6), it is clear that

$$n = \frac{m_p/m_f}{\ln(m_f/m_e)} = \frac{x}{\ln\left(\dfrac{1}{1-x}\right)} \qquad .. \qquad (7)$$

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where x is m_p/m_f. It is remarkable that the value of n should thus depend on the vehicle propellent fraction alone, and not directly on the density of the propellent. Selected values of $n(x)$ are given in Table I.

TABLE I

x	$n = \dfrac{x}{\ln\left(\dfrac{1}{1-x}\right)}$
0	1·00
0·2	0·898
0·4	0·783
0·5	0·721
0·6	0·655
0·8	0·497
0·9	0·390
0·95	0·317
1·0	0·0

A hypothetical example of a rocket designed under the constraints given might be an air-to-surface missile of fixed payload mass and payload diameter, weighing 1000 lb., of which 200 lb. is propellent. It is desired to maximize range, but instead speed is maximized, since range is an increasing function of speed. Now $x = 0{\cdot}2$, therefore $n = 0{\cdot}9$. Accordingly it is necessary to maximize the quantity $\rho^{0{\cdot}9} I_{sp}$. Consider next a single-stage ballistic missile with a specified payload designed to fit within a predetermined volume, such as a tube in the ground or in a ship. A fixed volume remains for propellent, so that it corresponds to the constraints given in this section. Such missiles normally have larger propellent fractions and smaller values of n. For example, if the missile weighed 10,000 lb., of which 7000 lb. was propellent, it would be necessary to maximize $\rho^{0{\cdot}59} I_{sp}$.

III. FULL MASS AND PART OF EMPTY MASS FIXED

The second case to be analysed is a rocket in which, as before, the empty mass comprises a fixed part m_{e1} and a part m_{e2} proportional to the propellent volume V. However, the full mass m_f will be fixed, while the propellent volume will not. Thus

$$m_{e2} = (k/\rho)\, m_p = (k/\rho)(m_f - m_{e1} - m_{e2}) \quad .. \quad (8)$$

or

$$m_{e2} = \frac{(k/\rho)(m_f - m_{e1})}{1 + (k/\rho)} \quad .. \quad .. \quad (9)$$

In modern design the ratio of chamber mass to propellent mass, k/ρ, is rarely greater than 0·15. Accordingly we may neglect k/ρ in the denominator of Equation (9). Therefore, differentiating (9), one finds

$$dm_{e2} = (-k/\rho^2)(m_f - m_{e1})\, d\rho$$
$$= -m_{e2}\, d\rho/\rho \quad .. \quad .. \quad .. \quad (10)$$

but

$$v = j \ln (m_f/m_e)$$

and since $dm_f = 0$, we have

$$dv = dj \ln \frac{m_f}{m_e} - j \frac{dm_e}{m_e} \quad .. \quad (11)$$

Because $dv = 0$, we have also

$$\frac{dj}{j} = \frac{dm_e}{m_e \ln (m_f/m_e)} = 0 \quad .. \quad .. \quad (12)$$

But $m_e = m_{e1} + m_{e2}$ and $dm_e = dm_{e2}$. Substitution of (10) into (12) therefore gives:

$$\frac{dj}{j} + \frac{d\rho}{\rho} \cdot \frac{m_{e2}}{m_e \ln (m_f/m_e)} = 0 \quad .. \quad (13)$$

Therefore, in this case

$$n = \frac{m_{e2}/m_e}{\ln (m_f/m_e)} \quad .. \quad .. \quad (14)$$

A typical single-stage ballistic missile might have $m_{e2}/m_e = 0{\cdot}5$ and $m_f/m_e = 5$. This would yield $n = 0{\cdot}31$. Thus, as is to be expected, when the missile is not volume-limited, the value of n is smaller; *i.e.*, the burnout speed depends less on the density ρ.

IV. FULL MASS AND PART OF EMPTY MASS FIXED, VARIABLE DIAMETER

In the third situation to be analysed, we wish to maximize the glide range of a missile. As before, part of the empty mass is assumed fixed; this might correspond to the payload. The full mass is also fixed. However, the third problem will differ from the second one considered in that the missile diameter is variable and the length fixed.

The glide range s is assumed proportional to the empty mass, a power of the burnout speed and inversely proportional to the frontal area; that is

$$s\Big]_{v}^{v'} \propto (m_e/d^2)\left\{v^p - (v')^p\right\} \quad .. \quad (15)$$

For pointed missiles it is found that p is about 0·6 since drag varies as $v^{2-0{\cdot}6} = v^{1{\cdot}4}$; for blunt-nosed missiles p is about 0·1 or less. From missile geometry we have:

$$d^2 \propto (m_p/\rho l_m)(l_m/l_p) \quad .. \quad .. \quad (16)$$

where l_m is the missile length and l_p is the length of the propellent chamber. It was specified that l_m was constant. The fundamental rocket equation gives

$$v = j \ln (m_f/m_e) \quad .. \quad .. \quad (2)$$

We now make the approximations that l_m/l_p is constant and that $(v')^p$ is much less than v^p; the second approximation can be roughly satisfied by adjusting the value of p. These approximations, together with equations (2), (15) and (16), yield

$$s \propto (m_e/m_p)\, \rho j^p \left[\ln (m_f/m_e)\right]^p \quad .. \quad (17)$$

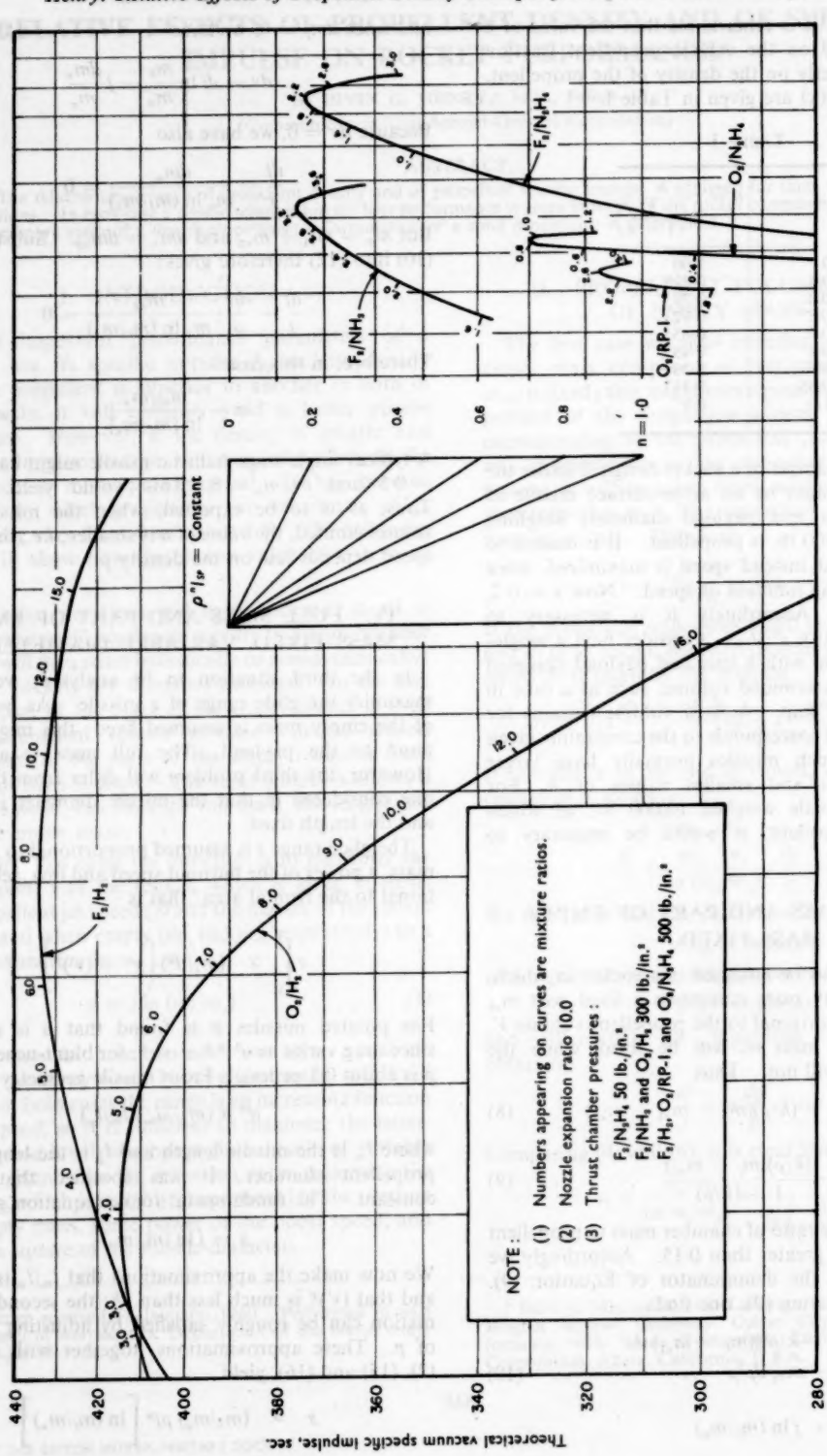

FIG. 1. Specific Impulse *vs.* Density of Cryogenic Propellents. *(See Fig. 2 for explanation of symbols).*

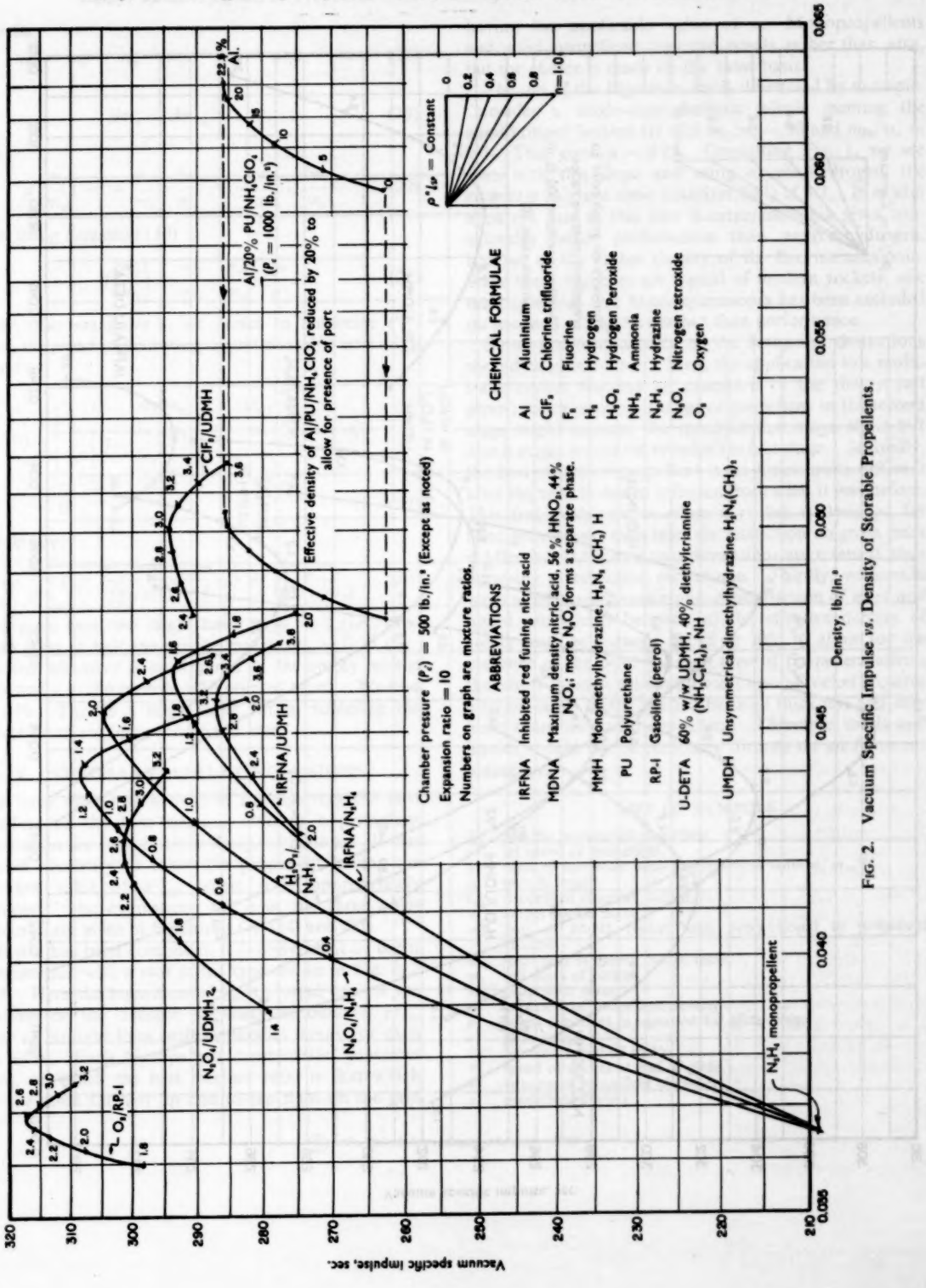

FIG. 2. Vacuum Specific Impulse *vs.* Density of Storable Propellents.

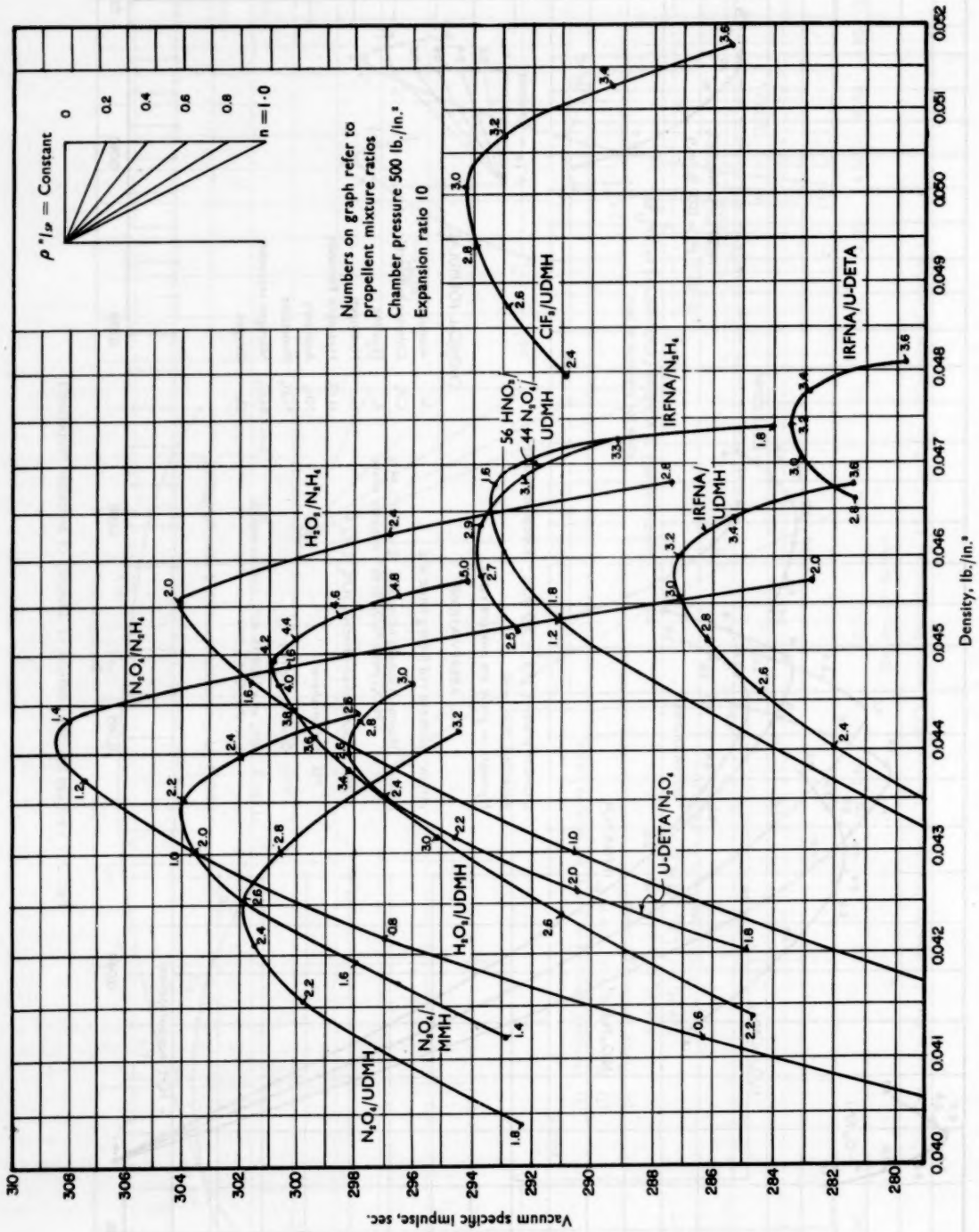

FIG. 3. Specific Impulse *vs.* Density of Storable Propellents (Enlarged Scale).

We write

$$m_p = m_f - m_e = m_f - m_{e1} - m_{e2} \quad .. \quad (18)$$

and

$$dm_e = dm_{e2} \quad .. \quad .. \quad .. \quad (19)$$

Thus

$$d\left(\frac{m_f}{m_e}\right) = -\frac{m_f}{m_e}\cdot\frac{dm_e}{m_e} = -\frac{m_f}{m_e}\cdot\frac{dm_{e2}}{m_e} \quad .. \quad (20)$$

or, on using Equation (10)

$$d\left(\frac{m_f}{m_e}\right) = \frac{m_f}{m_e}\cdot\frac{m_{e2}}{m_e}\cdot\frac{d\rho}{\rho} \quad .. \quad .. \quad (21)$$

Rather than maximize s, we prefer to maximize $s^{1/p}$. Doing this, and substituting Equations (18) and (21), we find

$$d(\ln s^{1/p}) = \left[\frac{1}{p}\left\{1 - \frac{\dfrac{m_f}{m_e}\dfrac{m_{e2}}{m_e}}{\dfrac{m_f}{m_e} - 1}\right\} + \frac{\dfrac{m_{e2}}{m_e}}{\ln\dfrac{m_f}{m_e}}\right]\frac{d\rho}{\rho} + \frac{dj}{j}$$

$$= 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (22)$$

Thus finally

$$n = \frac{1}{p}\left\{1 - \frac{(m_f/m_e)(m_{e2}/m_e)}{(m_f/m_e) - 1}\right\} + \frac{m_{e2}/m_e}{\ln(m_f/m_e)} \quad (23)$$

In a typical case one might have $m_f/m_e = 2$ (*i.e.*, propellent mass is half the missile mass), $m_{e2} = 0{\cdot}25$ (*i.e.*, propellent chamber is one-quarter of the empty missile mass), and $p = 0{\cdot}6$ (missile with pointed nose). We find $n = 1{\cdot}19$. This is a large value for n, reflecting the importance of reducing the frontal area.

V. SUMMARY AND CONCLUSIONS

We have presented a method for selecting the best propellent or the best mixture ratio of a propellent combination for each of three design situations. It was assumed necessary to select the propellent having the maximum value of $\rho^n I_{sp}$, where n is appropriately calculated. The expressions for n in the three cases considered are given in Equations (7), (14) and (23).

When n has been computed, it is convenient to use it in conjunction with a plot of the type shown in Fig. 1, 2 and 3. Here the logarithm of I_{sp} is plotted against the logarithm of the density, so that contours of $\rho^n I_{sp}$ appear as straight lines, with a parallel family for each value of n. Each bipropellent combination generates an arc, on which the best mixture ratio is that which gives the point furthest up and to the right on the grid having the applicable value of n. Monopropellents and solid propellents generate points rather than arcs, but the choice is made on the same basis.

The use of the Figures is easily illustrated by example. Consider a single-stage ballistic missile meeting the conditions of Section III with $m_f/m_e = 10$ and $m_{e2}/m_e = 0{\cdot}5$. This gives $n = 0{\cdot}22$. Consulting Fig. 1, we see that, with this slope and using oxygen/hydrogen, the optimum mixture ratio (oxidizer/fuel) is 5·7. It is also apparent that in this case fluorine/ammonia gives considerably better performance than oxygen/hydrogen, because of the higher density of the fluorine/ammonia. Since these numbers are typical of modern rockets, one must conclude that fluorine/ammonia has been excluded on grounds of logistics rather than performance.

Some practical aspects of the foregoing derivations should be pointed out. First, the application to a multi-stage rocket requires an extension of the theory just given; for example, a change of propellent in the second stage might increase the speed of that stage alone but also it might reduce the speed of the first stage. Secondly, the best choice of propellent is, at times, quite different after the vehicle design is frozen from what it was before. This frequently occurs when a rocket is designed for fixed gross mass; then later the restriction on gross mass is lifted but the motor case dimensions are retained, thus imposing a restriction on volume. Thirdly, we should warn against any oversimplified comparison of solid and liquid propellents because (*a*) the effective density of solids must be reduced by 15 to 30% to allow for the presence of the port, (*b*) each type of propellent affects the empty mass in a different way; a solid rocket requires a larger safety factor in chamber wall thickness and may leave more unburned propellent. Therefore solids and liquids should not be compared directly on an I_{sp} *versus* ρ diagram.

LIST OF SYMBOLS

I_{sp}	specific impulse of propellent.
j	jet speed of propellent.
k	ratio of hardware mass to propellent volume, m_{e2}/V.
l_m	missile length.
l_p	propellent chamber length.
m_e	empty rocket mass.
m_{e1}	part of empty rocket mass proportional to propellent volume.
m_{e2}	fixed part of empty rocket mass.
m_f	full mass of rocket.
m_p	propellent mass.
n	exponent of propellent density.
p	speed exponent in equation for glide range.
s	glide range.
v	burnout speed of rocket.
v'	speed of rocket at end of glide.
V	volume of propellent chamber.
ρ	propellent density.

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FIG. 3. Specific Impulse *vs.* Density of Storable Propellents (Enlarged Scale).

AN ENVIRONMENTAL CONDITIONING SYSTEM FOR A MANNNED SATELLITE*

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(Communication from The Martin Company)

ABSTRACT

A specific design is presented for an environmental conditioning system for use in a manned satellite designed to stay aloft for 60 days. Water is recovered by evaporative distillation and refrigeration, oxygen regenerated by electrolysis of water, carbon dioxide removed by use of lithium oxide (non-regenerative), and capacity for radiating 150,000 B.T.U./day from the satellite as a whole provided. With automatic control this system is suitable for use in an animal-carrying satellite. The weight is favourable when compared with other systems that have been suggested, and well within the conditioning allowance of a 5000-lb. manned satellite.

I. INTRODUCTION

THE problem of controlling the environment in a manned satellite, which is to remain aloft for more than a few days, is a difficult but essential one. It is necessary to remove heat, control humidity, remove carbon dioxide and replenish oxygen. Many relatively simple ways of doing this are known, but they universally suffer from the defect of high weight and power requirements.

A hypothetical case has been chosen to permit the making of calculations for a system of control which is applicable to a wide range of purposes. We have chosen the case of a satellite with an approximate 90-minute orbit carrying a man and designed to stay aloft for 60 days before recovery. Such a satellite would not need to weigh over 5000 lb. and is within the limits of accomplishment in the near future by use of the Titan missile as the basic booster. Power will be supplied to the satellite by solar cells with an average daylight level of approximately 1½ kW. A sun-seeker is to be used for solar cell orientation and part of their output power is stored in batteries for use while the satellite is in shadow. The power from the solar cells must satisfy requirements for all purposes including control, telemetering and environmental conditioning. Peak loads, as for example communications and command data telemetering, are satisfied with the assistance of stored battery power. To meet requirements, an estimated 120 ft.² of cell surface area will be needed.

Heat removal is one of the more serious problems in a satellite, as not only the cabin but the electronics compartment must be cooled and the heat radiated to space. The design presented here is capable of handling 150,000 B.T.U./24 hr. in the life-ambient range of temperature.

The fact that the satellite is manned simplifies the control system and increases general reliability. The same system could be used for an animal-carrying biological satellite with the addition of a number of automatic control devices.

Details on the major parts of the system are given below.

II. ATMOSPHERE CONTROL SYSTEM

1. DISCUSSION

The atmosphere used in this system will consist of 144 mm. of oxygen pressure, 130 mm. of helium pressure, and not over 5 mm. of carbon dioxide pressure. The water vapour pressure is variable. Table I gives the quantity of various products a man will produce or need[1–3] under light activity in 24 hours.

TABLE I

Product or Requirement	Rate of Production or Use	Remarks
Water	6·6 lb./day	Produced (less required)
Oxygen	2·2 lb./day	Required
Carbon Dioxide	2·2 lb./day	Produced
Methane ..	0·3 l./day	Produced
Heat (biological)	12,000 B.T.U./day	Produced
Heat	150,000 B.T.U./day	Total to be radiated from satellite including all sources.

The problem of the oxygen supply was examined quantitatively. The use of gaseous oxygen leads to a very high mass requirement (about 300 lb. for the oxygen and tanks alone) for a 60-day satellite, largely due to the mass of the tanks. Utilization of liquid oxygen is quite feasible using lightweight vacuum-insulated containers at about 2 atm. pressure. The evaporation rate under such conditions should not exceed the rate at which oxygen is needed initially, and with the best insulation external heating would be needed to maintain the necessary evaporation rate. A mass of 150 lb. total is estimated for the oxygen supply if this system is used.

It is apparent that a man will produce, because of combustion of foods, more water than he consumes. It is desirable to reclaim essentially all water available. This is discussed in a later section. Once recovered, this water over and above that needed for drinking purposes is surplus and available for decomposition to recover the oxygen. This would result in a substantial

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weight saving in the atmosphere system. The only oxygen that need be lost to the system is that going into carbon dioxide. This amount minus the amount produced by food combustion is about 60 lb. We need, therefore, to carry about 75 lb. of water at takeoff in order to have a balanced system for 60 days and an adequate safety factor. If waste water is not reclaimed, 300 lb. of water in addition to 150 lb. of oxygen plus container would have to be carried.

A system has been devised whereby the oxygen needed is produced by electrolysis of water. It utilizes a low-speed rotating cell where the "centrifugal force" keeps the gas and water separate.

In order to conserve battery power, the electrolysis cell is run only during the time the satellite is in sunlight and direct power from the solar cells is available. Enough oxygen is produced and stored during this time to last while the satellite is in the Earth's shadow. The hydrogen formed in the electrolysis can be vented to space or used to replenish the cooling system gas if it is used as heat transfer medium.

Another problem is that of keeping the carbon dioxide content of the atmosphere below 5 mm. partial pressure. For a trip of this duration and with only solar power available, the use of a regenerative system, such as that based on calcium oxide, is not economical. Lithium oxide is the best non-regenerative agent available from the standpoint of mass. To remove the carbon dioxide produced, a total of 80 lb. of lithium oxide will be required. A 25% safety factor has been added, making a total of 100 lb.

Heat removal from the cabin or animal cells proves to be the controlling factor from the standpoint of air re-circulation. In order to remove the 12,000 B.T.U./24 hr. produced by a man or animals, an air circulation rate of 80 ft.³/min. is required if air is to enter at 58° F. (14·4°C.) and leave at 80°F. (26·7°C.). This would be about 80% of available air exchanged each minute if a compartment of 100 ft.³ is used. Since the removal of carbon dioxide and water does not need to proceed at such a high rate (with respect to air flow), an economy in power and weight is effected through by-passing 70% of the air flow around the carbon dioxide and water removal system.

The liberation of body gases presents a problem. The major constituent is methane and in 30–40 days a concentration would build up that could reach the lower explosive concentration of 5% by volume of the cabin atmosphere. It is therefore necessary to remove the methane. Unfortunately this is not simple as it is an extremely stable molecule. One possible way is to rapidly and completely exchange the entire atmosphere at the end of 30 days. However, by use of a small platinum coil heated electrically to 500°C. it is possible to burn the methane to carbon dioxide and water. This is carried out just before the gas enters the blower and after the oxygen content has been raised to 144 mm. partial pressure. Any hydrogen present would also be oxidized. As methane generation is slow and a concentration of several percent is tolerable, it is not necessary to cycle the entire atmosphere stream through this step. The rate of air flow can be maintained through this cell at 2 ft.³/min. and the remainder of the atmosphere 97·5%) by-passed. The power requirement and heat produced with this low flow rate in a cell using a heat exchanger is quite small, approximately 10–15 W.

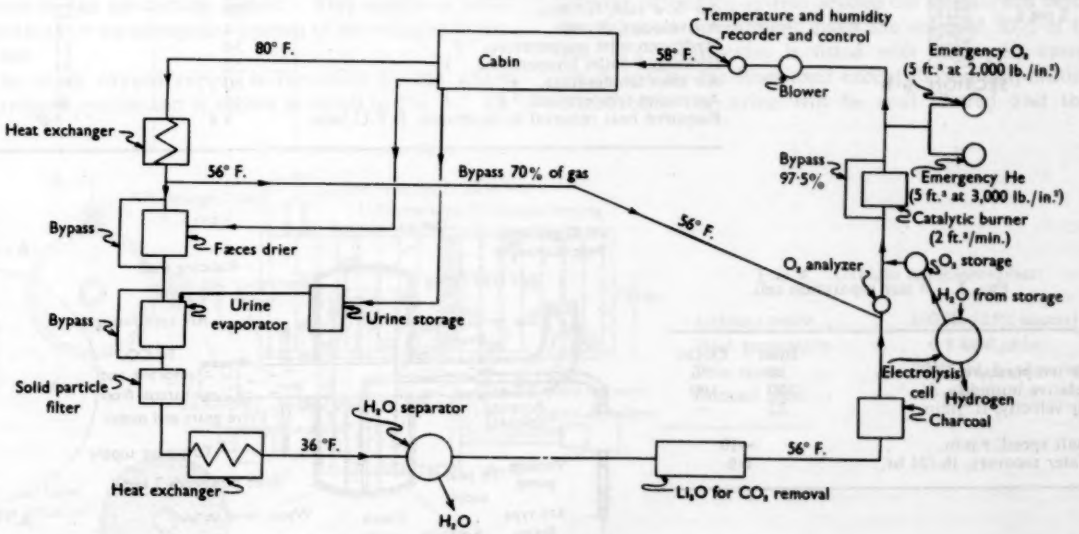


FIG. 1. Atmosphere system.

Pressure: 5 lb./in.²
Humidity: 40–50%.

Circulation: 24 ft.³/min. in Condenser, 80 ft.³/min. total.
Composition: O_2 144 mm., He 130 mm., CO_2 7 mm. (max.), H_2O variable.

Other body gases (mainly sulphur compounds) are removed by a small activated charcoal bed.

Oxygen and nitrogen free-radicals formed by space radiation and particle collisions with the cabin gases could prove a health problem if allowed to concentrate. However, products, mainly ozone, are destroyed both by the activated charcoal-silver bed and by the catalytic burner. It is likely that some of the ozone will react with the methane while still in the cabin volume. If helium is used instead of nitrogen, nitrogen free-radicals will not be of concern. This might simplify environment control.

2. Description of the System as Shown in Fig. 1.

A flow diagram of the specific system is shown in Fig. 1. Gas of proper composition and temperature enters the compartment under flow rate control; It mixes with the gas in the chamber and leaves by screened vents. The gas temperature will rise from 58°F. (14·4°C.) to 80°F. (26·7°C.) while passing through the compartment. The temperature, humidity, and oxygen content of the incoming air is automatically monitored and is subject to automatic or manual control at the operator's discretion. The 80°F. air forced out of the cabin passes through a heat exchanger (Fig. 2) which cools it to about 56°F. (13·3°C.). At this point, approximately 70% is by-passed and returned to the cabin without moisture or carbon dioxide removal.

The remaining 30% of the gas at 56°F. is passed through another heat exchanger where it is cooled to 36°F. (2·2°C.). This results in a mixture of saturated air and water droplets which is passed into a centrifugal separator (Fig. 3) where water is removed and sent to storage. A vortex separator using "moisture lock"

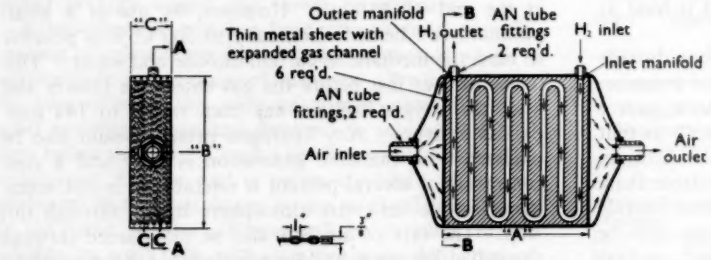
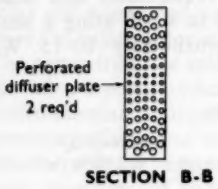


Fig. 2. Heat exchangers.



	Air Cooler	Water Condenser
Dimensions "A" and "B," in.	18	12
Dimension "C," in.	6	8
Hydrogen flow rate, ft.³/min.	60	60
Air flow rate, ft.³/min.	80	22
Air velocity, ft./min.	4	2
Hydrogen inlet temperature, ° F.	34	23
Hydrogen outlet temperature, ° F.	52	34
Air inlet temperature, ° F.	80	56
Air outlet temperature, ° F.	56	36
Required heat removal by hydrogen, B.T.U./min.	9·4	5·0

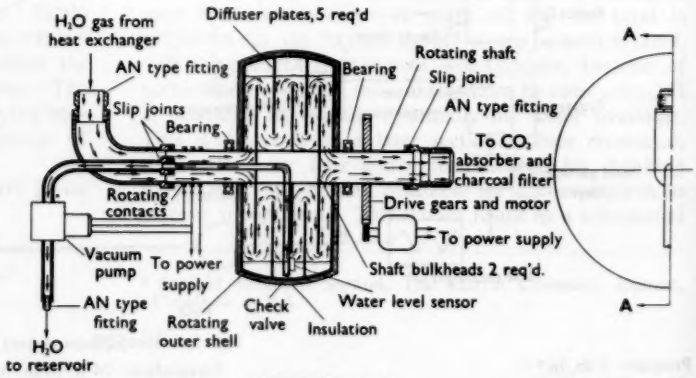


Fig. 3. Water separation cell.

	Inlet	Outlet
Air temperature, ° F.	36	36
Relative humidity, %	100	100
Air velocity, ft.³/min.	22	—
Shaft speed, r.p.m.	~10	
Water recovery, lb./24 hr.	6·6	

filler may be used in place of the centrifuge. The saturated gas at 36°F. is then passed into the carbon dioxide removal chamber (Fig. 4). It flows first around the outside in a double wall to keep the absorption chamber cool and to pick up heat (to reduce the relative humidity). The amount of heat liberated from he overall reaction:

$$Li_2O + CO_2 = Li_2CO_3$$

is sufficient to heat the normal flow of gas to about 56°F. The lithium oxide is in granular form and is separated into 2-in. thick layers by perforated plastic disks to minimize caking and give relatively unimpeded gas flow at all stages of the tube's life. The air will contain the moisture necessary for the above reaction to take place. The rate of the reaction:

$$Li_2O + H_2O = 2LiOH$$

is slow at medium temperature, and neither the carbonate or hydroxide is hydroscopic to any extent. This system will keep the carbon dioxide at a low level, certainly well below the 5 mm. maximum.

The exit gas flows through a charcoal bed to remove other gases. From here it joins the by-pass line and the joint stream passes through the automatic oxygen analysis and make-up system. Most of the gas then passes through the blower, by-passing the catalyst chamber for methane removal. The 2·5% which does pass through this chamber will pass over a coil heated to 500°C. and the methane and any hydrogen that has leaked into the system oxidized. Possibly the cold oxidation of methane and hydrogen can be brought about by a catalyst, such as divided platinum, combined with one or another radioactive isotope. An emergency oxygen and helium supply ties in just before the blower in the air cycling system. This supply is sufficient to allow an emergency descent of the vehicles to the surface.

The main oxygen supply is furnished by the water electrolysis system and is shown in detail in Fig. 5. To supply the 2·2 lb./day oxygen which a man needs will require the electrolysis of about 2·5 lb. water. In practice, about 2·86 lb. water per 24 hours will be electrolyzed. The electrolysis of water can be carried out at approximately 95% efficiency at moderate current densities according to date available[4]. Approximately 190 W. constant power is required. However, no solar power will be available when the satellite is in shadow; this is about one-third of the time for an equatorial orbit of near 90 minutes. To conserve battery capacity requirements, it is planned to stop electrolysis during dark periods; therefore about 285 W. for 15 hr. a day will be required. The actual cycle will consist of shutdown for approximately 30 min. and operation for 60 min. out of every 90 min. The oxygen needed during the periods of shutdown will be stored at one atmosphere pressure in the low-mass shell surrounding the cell. Hydrogen liberated at one of the electrodes will either be vented to space or used in the cooling system. The electrolyte will probably be acid rather than the more commonly used alkali because of emulsion problems. This point, however,must be checked experimentally.

The problem of rotary gas seals for this cell was solved several years back by refrigerant pump manufacturers. A loss of one part in 10^4 per year is considered excessive for such seals. However, it appears entirely feasible to do away with the rotating cell and use glass fibre for holding the electrolyte. Some research needs to be done on such cells relative to free-fall environment.

One part of the cycle that will be operated manually and intermittently is the recovery of moisture from urine (1500 ml./day). Fæces and urine will be placed in separate plastic containers and the urine returned into the system as shown in Fig. 1. The urine is introduced into the part of the system beyond the by-pass and before the exhaustive treatment given the residual 30% of the gas. The evaporator is fitted with a by-pass system which will be the lines used except during evaporation. Periodically the urine will be well filtered and then

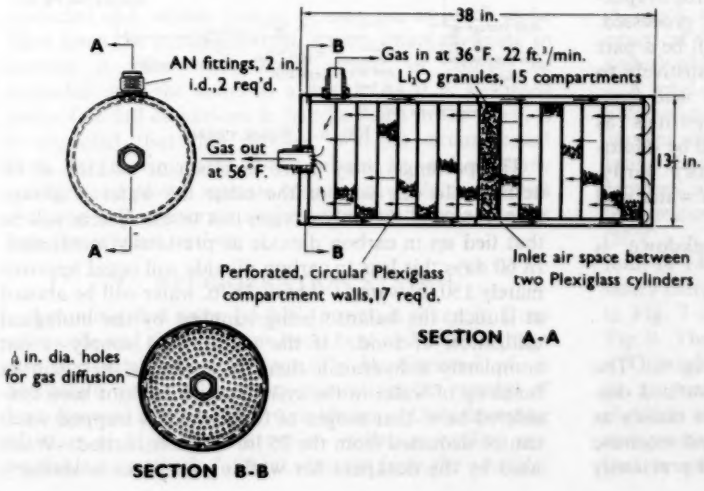
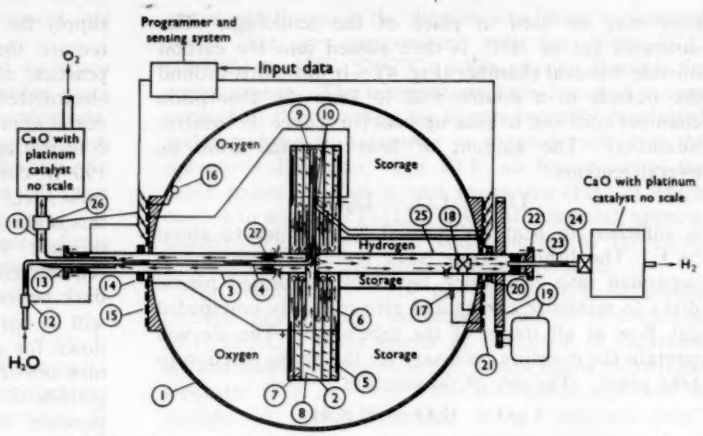


FIG. 4. Lithium oxide container.

Lithium oxide	..	100 lb. (25% excess)
Heat generation	..	0·8 kcal./min.
Free space	..	200% of Li_2O volume
Contact time	..	5 sec.



SECTION VIEW

1	Fibreglass sphere	11	Oxygen outlet tube	18	Hydrogen demand valve
2	Electrolysis cell	12	Demand valve	19	Motor and drive gears
3	Oxygen shaft	13	Rotary seal	20	Bearing
4	Water supply tube	14	Bearing	21	Rotating contacts
5	Electrolysis plates	15	Rotating contacts	22	Rotary seals
6	Separator plates (bottom perforated)	16	Pressure sensor	23	Hydrogen supply tube
7	Separator plates (bottom perforated)	17	Pressure sensor	24	Check valve
8	Water and acid			25	Hydrogen shaft
9	Stationary agitator			26	Oxygen demand valve
10	Water and acid level sensors			27	Perforated support

FIG. 5. Electrolysis cell.

Electrolysis rate	..	2·6 lb. water/24 hr.
Power requirements	..	190 W. constantly or 285 W. for two-thirds of the time
System speed	..	~10 rev./min.
Sphere diameter	..	21 in.

sprayed slowly into the evaporator. Urine bacteria can be controlled by a trace of silver salt. This evaporator consists of a series of perforated silver-plastic baffle-plates arranged so that considerable turbulance results. The droplets are broken up into fine spray which will evaporate rapidly leaving the filterable solid materials in the chamber. The silver-plastic baffle plates can be periodically replaced as solids build-up dictates. These plates will have a mass of some 4 oz. per set. As the fæces normally contains little moisture (about 150 ml./24 hr. is produced) it appears simpler to discard it than to remove the moisture. If necessary this moisture could be recovered but at some mechanical difficulty. The air in this part of the system will normally bypass the evaporator except for periods when urine is being processed. The moisture from the urine and/or fæces will be a part of the 30% of the gas which is treated exhaustively to remove moisture, carbon dioxide, gaseous acid constituents, micro-organisms, and other impurities as indicated in the flow diagram. The result will be odourless and sterile air and water of satisfactory purity. The water will be the equivalent of distilled water and suitable for drinking or electrolysis.

A weight and power requirement breakdown is summarized in Table II.

III. WATER CYCLE

The overall cycle for water is shown in Fig. 6. The water is forced on demand by the gas pressurized diaphragm into the cabin. The water will leave mainly as vapour or as urine. The recovery of urine and moisture in the air as liquid water has been discussed previously as it is integrally connected with the atmosphere cycle. After leaving the centrifugal separator the water passes through a silver-charcoal bed to remove gas and any materials carried over and to destroy bacteria. Next, the treated water goes to storage. The storage tank also holds replaceable activated charcoal inserts to further remove dissolved gas. The water produced is completely satisfactory for consumption although there is admittedly a psychological problem in so convincing a man.

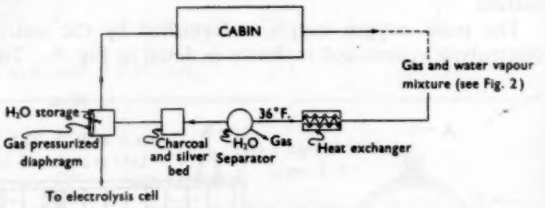


FIG. 6. Water system.

The passenger may drink as much or as little as he desires. In one way or the other the water is always recovered and the only oxygen lost to the system will be that tied up in carbon dioxide as previously mentioned. In 60 days this loss to carbon dioxide will equal approximately 150 lb. water. About 75 lb. water will be aboard at launch, the balance being supplied by the biological utilization of food. If the man's food supply is not completely dehydrated, there is a further net gain or build up of water in the system which has not been considered here, that weight of food which is trapped water can be deducted from the 75 lb. of water carried. Water used by the occupant for washing purposes is similarly

TABLE I.—*Conditioning System*

Equipment	Mass, lb.	Power, W.
Atmosphere Control Equipment		
Urine storage plus replaceable filters	12	—
Fæces storage	4	—
Urine evaporator	5	—
Solid particle filter (including glass wool)	4	—
Heat exchanger (air cooling)	12 (15 for He)	—
Heat exchanger (water condensation)	8 (12 for He)	—
Centrifugal separators	5	Same shaft as electrolysis cell
Battery to support conditions system	10	—
Carbon dioxide absorption	110	—
Charcoal bed	5	—
Electrolysis cell	8	*285 max. (zero min.)
Initial charge of water in electrolysis cell	10	—
Motor to run cell	1	25
Combustion chamber (catalyzed)	2	20
Blower	5	125
Emergency Oxygen Tank plus Oxygen gas	5	—
Emergency helium tank plus helium	10 (50 for He)	—
Miscellaneous lines, controls, regulators, etc.	18	20
	234 (281 for He)	475 max. 380 average
Water System		
Contact chamber, heat exchangers and centrifugal separators	Mass and power are entered under the atmosphere system.	
Charcoal-silver bed	5	—
Water storage chamber, etc.	10	—
Water supply, initial	75	—
	90	—
Heat Radiator System		
Heat exchange gas blower	5 (8 for He)	265 (575 for He)
Black body radiator	37 (50 for He)	—
Bladder valves w/control valves, lines, etc.	8 (20 for He)	1
10 lb./in.^{2} compressor and 1 ft.^{3} tank (supplies 1 ft.^{3} of gas per hour)	4	10
Fittings, lines, connectors, etc.	50	—
	104 (132 for He)	276 (586 for He)
Totals	428 (503 for He)	Max. 751 (1061 for He) Average 656 (966 for He)

* No power is required in shadow as cell is shut down to save battery weight. Time in shadow is assumed as being one-third of time.

re-cycled and, within reason, he may use as much as he likes since the cooling system is more than adequate to recover it. The wash water would, of course, be re-cycled with the urine as a flush. As it is awkward under free fall conditions to handle fluids in the open, it is expected that this problem will be circumvented both by moving free fluids about in polyurethane sponges and plastic squeeze containers. Water droplets escaping will evaporate or be swept by the moving air into the exhaust outlets.

IV. REFRIGERATION SYSTEM

Assuming that over 1·5 kW. power is taken in from solar cells and that this plus body heat and solar leakage into the satellite must eventually be disposed of, it becomes a matter of necessity to consider the method of heat removal early in the system design. Only dumping of mass at high temperature or radiation cooling are available as methods for heat removal from the satellite. A quick calculation shows that mass-dumping is impractical because of the quantity of mass (preferably water) which would have to be carried.

A total of as much as 150,000 B.T.U./day (a considerable safety factor is contained in this number) from all sources must therefore be removed by radiation. Calculation shows that to radiate this amount of heat to space requires about 110 ft.^{2} of black body surface if the temperature differential from 5°K. in space to 353°K. (80°C.) in the electronic chamber could be utilized. A total of 142 ft.^{2} of radiator is used to provide adequate safety margin. The design of the radiator itself is shown in Fig. 7 and 8 and the overall refrigeration system in Fig. 9. The Appendix gives the thermodynamic details.[5,6]

The radiator consists of ten separate channels running in alternate spirals about part of the hull. These channels all separate where the gas leaves the manifold and passes through the bladder valves. This is done so that in case of meteor holing or leakage the channel

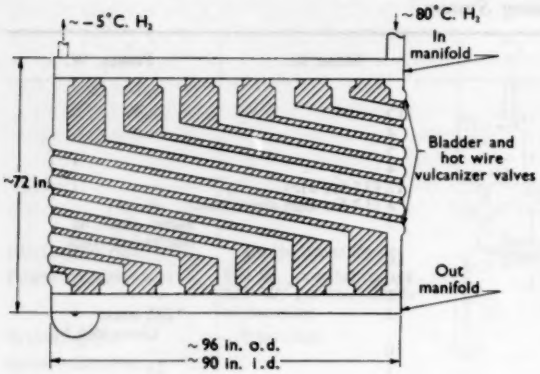


FIG. 7. Hydrogen cooling system radiator.

NOTES

1. Radiator channels ~0·01 in.
2. Aluminium or stainless steel sheet.
3. Channel width ~4 in.
4. Channel depth ~1 in.
5. Channel spacing ~0·5 in.
6. Turns per channel ~1.
7. Channel length ~30 ft.
8. Channel volume ~1440 in.³
9. Manifold rings ~96 in. o.d., ~90 in. i.d., length 10 in.
10. Bladder valve housing length ~3 in., diameter ~2 in.
11. Inlet diameter ~4 in., outlet diameter ~3 in.
12. Gas flow per channel ~6 ft.³/min.
13. Gas flow total ~60 ft.³/min.
14. Inlet gas temperature ~80° C.
15. Outlet gas temperature ~−5° C.
16. Gas stay time ~8 sec.

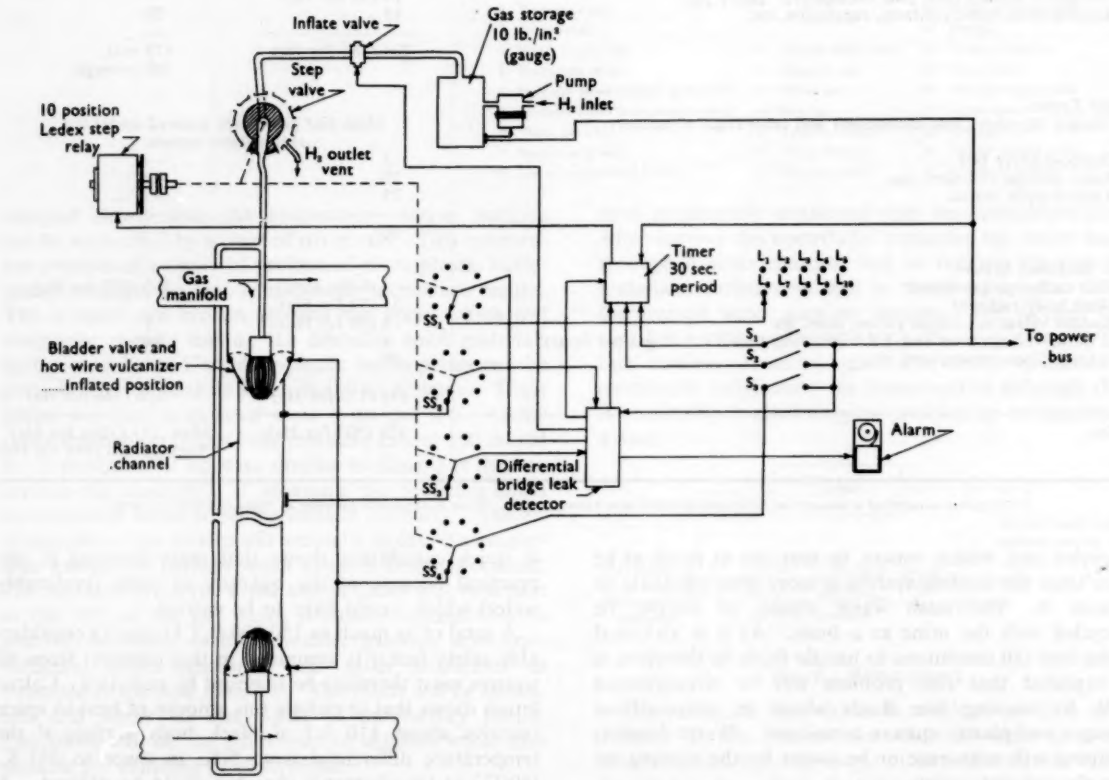


FIG. 8. Radiator leak detector and seal-off system.

NOTE

Variation from pressure/temperature norm relation of ±1% must be detectable. ±0·3% is believed to be obtainable. This is based on hydrogen replenishment rate of 0·0015 g./sec. available from electrolysis.

LEGEND

L_1–L_{10}	Leak warning system.
S_1	Timer hold switch.
S_2	Timer override switch.
S_3	Vulcanizer switch.
SS_1	Warning lamp switch.
SS_2	Channel pressure switch.
SS_3	Radiation temperature switch.
SS_4	Vulcanizer selector switch.

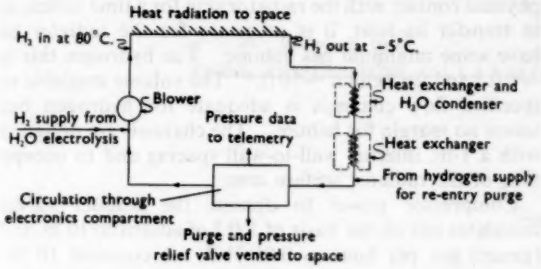

Heat radiation to space
H_2 in at 80°C.
H_2 out at −5°C.
Heat exchanger and H_2O condenser
Blower
H_2 supply from H_2O electrolysis
Pressure data to telemetry
Heat exchanger
From hydrogen supply for re-entry purge
Circulation through electronics compartment
Purge and pressure relief valve vented to space

FIG. 9. Hydrogen system.

Pressure	15 lb./in.²
Flow through radiator	60 ft.³/min.
Radiator surface area	142 ft.²

involved can be automatically shut off from the system before much gas has been lost. The system should be operable even if three of the ten channels are put out of commission. The radiator is made of two sheets of thin flexible metal brazed together to form gas channels which can be inflated by compressed gas.

The electronics compartment is designed to run at 80°C. exit gas temperature. Gas at ~ 15°C. is diffused in at one side and a blower forces out the heated gas at the other side. The entire system will operate at 760 mm. in contrast to the 275 mm. pressure in the cabin atmosphere system.

The gas at 80°C. will be cooled by radiation to −5°C., whereupon it passes through the two cabin heat exchangers and then into the electronic and mechanical equipment compartment completing the cycle. As demand varies, depending on whether the satellite is in the sunlight or shadow, the coolant gas flow must be variable.

The choice of a heat transfer gas was considered carefully and hydrogen chosen for several reasons. The first is its high heat capacity and low viscosity. The second is the fact that it can be replenished in case of leakage from the hydrogen produced in the electrolysis cell. A major disadvantage is the explosion hazard. The explosive limits are from about 5% to 94% by volume of hydrogen in oxygen. However, hydrogen has been used for many years as a generator coolant with no serious accidents. If helium is used as the coolant, a marked penalty is paid in blower power requirement; this calculates to be in excess of a 100% increase. Leakage of the hydrogen/oxygen into the atmosphere/coolant systems represents the greatest hazard, and it is partly for this reason (as well as for methane and ozone removal) that the platinum catalyst chamber was added to convert these gases to water and carbon dioxide. Short of major destruction of some part of the system, the use of hydrogen as a heat transfer agent should present no danger. Detection equipments exist which could act, in case of excess leakage, to dump the hydrogen and replace it from the helium tank. This would allow time for descent from orbit at any selected point along the orbit path.

V. CONCLUSION

As shown in Table II, the weight of the system designed for a 60-day manned satellite is relatively low when compared with other systems. Some developmental research is required to make it operable, but only minor changes should be necessary to obtain a satisfactory system. While designed for one man, the addition of automatic controls would make it suitable for an animal-carrying satellite. Further, it can be readily scaled for any crew size desired.

For trips longer than about three months, the use of a calcium oxide or equivalent regenerative system for carbon dioxide removal is definitely indicated. Such a system requires considerable power and heat dissipation, as the temperature of most carbonate decompositions is about 1000°C.

It appears reasonable to assume that from reasons of safety helium as a heat exchange gas would be more attractive for a manned vehicle than hydrogen. However, as indicated in the Appendix, a mass and power penalty is involved in its use. If mass is the deciding factor, then hydrogen is apparently the most efficient heat transfer substance available.

When the vehicle descends from orbit, the hydrogen coolant is dumped overboard along with the radiator system. During the descent, cabin cooling is maintained by evaporation of the water reserve. It is assumed that the thermal flux into the cabin from re-entry heating is reasonably low.

APPENDIX

For long-term operation, radiation cooling is the only feasible method of removing heat from a body in a vacuum. The familiar Stefan-Boltzmann law ($E = G\epsilon T^4$) allows a simple solution for a radiator whose emissivity is known. As deep space has a radiation temperature of about 5°K., it can be assumed to be a perfect sink for all but very low black body temperatures.[4]

The temperatures assumed for the heat exchange gas are inlet 80°C., outlet −5°C. As the radiator must be at a lower temperature than the gas to absorb heat, it was necessary to arrive at some temperature which would average the radiator black body temperature. By treating the surface in infinitesimals, an average temperature of ~30°C. was arrived at. Due to unknowns, this value is likely off by ± 10%. However, sufficient allowance has been made to cover such errors.

An emissivity ϵ of 0·8 was chosen as this is readily achievable. Using a G of $0{\cdot}482 \times 10^{-12}$ B.T.U./ft.²/sec./(°R.)⁴, the average radiation level was found to be $0{\cdot}482 \times 10^{-12} \times 5{\cdot}86 \times 10^{10} \times 0{\cdot}8 = 2{\cdot}26 \times 10^{-2}$ B.T.U./ft.²/sec.

The B.T.U. input from all sources is assumed as 150,000 B.T.U./24 hr. However, as most of this arrives during the sunlight period, a basic radiation capacity of 2·5 B.T.U./sec. is assumed necessary. This requires a surface radiating area of approximately 110 ft.² Adding 30% to this to allow for loss of gas channels due to

meteor puncture or mechanical leaks gives a total surface area of approximately 142 ft.²

The radiator material chosen is black surfaced aluminium alloy approximately 0·01 in. thick with an overall mass of ~0·13 lb./ft.² This gives a total skin mass of ~37 lb. Ends, fittings, bladder valves and transducers will be about 50 lb. additional mass, bringing the total to 87 lb.

To transfer the peak of 2·5 B.T.U./sec. to the radiator at an inlet temperature of 80°C. and an outlet of −5°C. requires a flow rate at constant volume at 1 atm. of 0·31 lb./min. hydrogen or 1·27 lb./min. helium. For the stated conditions and 30°C. average skin temperature, this is 59 ft.³ hydrogen or 127 ft.³ helium. Air H.P. is given by $\frac{\text{ft.}^3/\text{min.} \times 46/\text{in.}^2 \times 144}{33{,}000}$. For hydrogen at 0·5 lb. pressure head this is $59 \times 0{\cdot}5 \times 0{\cdot}435 \times 10^{-2} = 0{\cdot}128$ H.P., and for helium $127 \times 0{\cdot}5 \times 0{\cdot}435 \times 10^{-2} = 0{\cdot}276$ H.P. Assuming a 50% efficient impeller and a 50% efficient motor, a peak power requirement of 380 W. = 0·276 H.P. Assuming a 50% efficient impeller and a 50% efficient motor, a peak power requirement of 380 W. for hydrogen or 825 W. for helium is required. However, based on the average heat load of 1·74 B.T.U./sec., the average blower power requirement is 265 W. for hydrogen and 575 W. for helium. To insure proper physical contact between the gas and radiator inner surface, it is necessary that turbulent flow exist. This, in turn, slightly raises the blower power requirement due to increased back pressure. As the gas must remain in physical contact with the radiator skin for a time sufficient to transfer its heat, it is necessary for the radiator to have some minimum gas volume. For hydrogen this is ~6 ft.³ and for helium ~10 ft.³ The volume available in specified flow channels is adequate for hydrogen but leaves no margin for helium. The channels are designed with a 1-in. internal wall-to-wall spacing and to occupy 80% of the radiator surface area.

Compressor power to operate the bladder valves calculates out on the basis of 1 ft.³ of adiabatic 10 lb./in.² (gauge) gas per hour as 0·01 H.P. A constant 10 W. consumption is assumed. A compressor and tankage mass of 4 lb. is assumed as obtainable for this system. 8 lb. mass is assumed for the valves, tubes, and Ledex-type stepper which controls the gas flow to and from the bladder valves. The Ledex valve relay will use ~100 W. per step.

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CORRESPONDENCE

(In these letters to the Editor, some passages have necessarily been omitted)

PROFAC Power Sources: a Correction

SIR,

The following two corrections should be made to my paper[1]. In Section II.5 (p. 119), third paragraph, the fourth sentence should read: "Such a power source with auxiliary equipment would weigh about 42 tons with the present state of the art and perhaps as low as 20 tons with the expected development of nuclear power sources in 10 years. . . ."

The correct masses are *42 tons* and *20 tons*, not *11 tons* and *2 tons* as given in the paper.

Yours very truly,

S. T. DEMETRIADES.

Northrop Corporation, NORAIR Division,
1001 East Broadway, Hawthorne,
California, U.S.A.

17 *November*, 1959.

REFERENCE

1. S. T. Demetriades, *J.B.I.S.*, 1959–60, **17**, 114.

Earth Satellite Problem[1]

SIR,

It is required to place a given payload into a circular orbit of a specified radius by means of a two-stage "all-propellent" rocket with *minimum* propellent consumption. It is assumed that the Earth is non-rotating and spherically symmetrical; that both stages burn instantaneously, one at the surface of the Earth and the second at satellite altitude; and that the flight is in vacuum. The payload ratio (ratio of total propellent mass to payload mass) is expressed by:

$$R = e^{F} \quad .. \quad .. \quad .. \quad (1)$$

where

$$\left.\begin{aligned} F &= \frac{1}{c_1}\sqrt{\frac{\mu}{r_1}}\left[\sqrt{2(1-\rho_r)(1-\rho_c^2)} + \rho_c\sqrt{\rho_r}\right] \\ \text{and} \quad \phi &= \tan^{-1}\left[\sqrt{(\rho_r/\rho_c)^2 - 1}\right] \end{aligned}\right\} \quad (2)$$

ϕ being the direction angle from horizontal of first-stage thrust (launch direction).

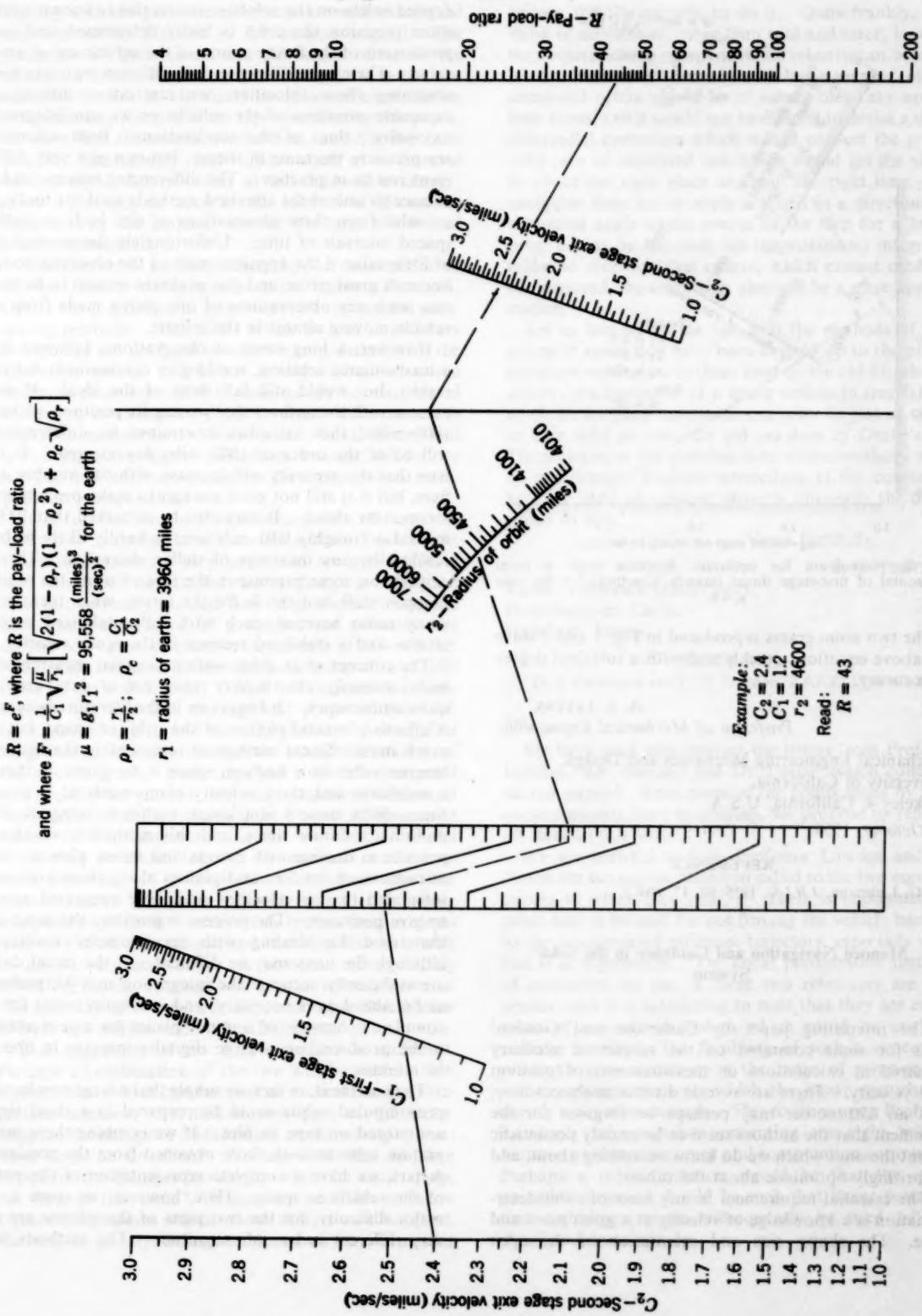
R – Pay-load ratio
4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100 200
C2 – Second stage exit velocity (miles/sec)
3.0 2.5 2.0 1.5 1.0
r2 – Radius of orbit (miles)
7000 6000 5000 4500 4100 4010
C1 – First stage exit velocity (miles/sec)
3.0 2.5 2.0 1.5 1.0
C2 – Second stage exit velocity (miles/sec)
3.0 2.9 2.8 2.7 2.6 2.5 2.4 2.3 2.2 2.1 2.0 1.9 1.8 1.7 1.6 1.5 1.4 1.3 1.2 1.1 1.0
R = e^F, where R is the pay-load ratio
and where F = 1/C1 √(μ/r1) [√(2(1 − ρr)(1 − ρc²)) + ρc √ρr]
μ = g1 r1² = 95,558 (miles)³/(sec)² for the earth
ρr = r1/r2; ρc = C1/C2
r1 = radius of earth = 3960 miles
Example:
C2 = 2.4
C1 = 1.2
r2 = 4600
Read:
R = 43

FIG. 1. Nomogram for optimum payload ratio, R, for the case $\rho_r \leq \rho_c$

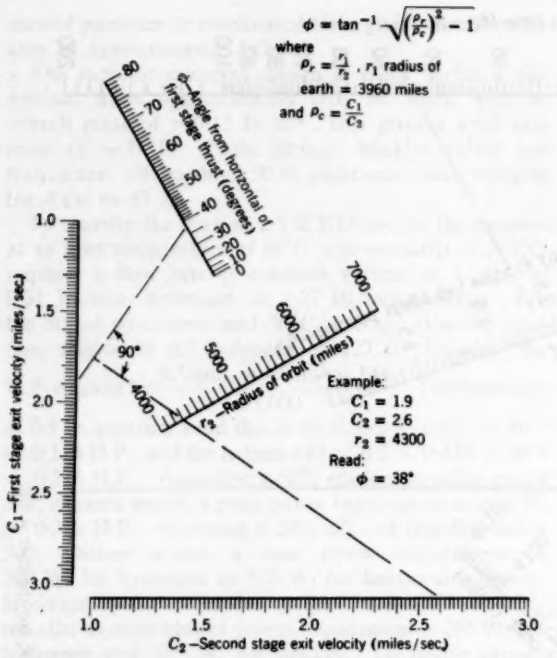

FIG. 2. Nomogram for optimum direction angle ϕ from horizontal of first-stage thrust (launch direction) for the case $\rho_c \leqslant \rho_r$

The two nomograms reproduced in Fig. 1 and 2 solve the above equations quickly and with a sufficient degree of accuracy.

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Professor of Mechanical Engineering.

Mechanical Engineering Mechanics and Design,
University of California,
Berkeley 4, California, U.S.A.

14 *October*, 1959.

REFERENCE

1. G. Leitmann, *J.B.I.S.*, 1959–60, **17**, 194.

Manned Navigation and Guidance in the Solar System

SIR,

The interesting paper by Cashmore and Gordon[1] calls for some comment on the subject of accuracy required in calculations or measurements of position and velocity. There are several distinct problems here, and an astronomer may perhaps be forgiven for the comment that the authors seem to be unduly pessimistic about the ones which we do know something about, and surprisingly optimistic about the others!

The essential requirement in any form of orbit determination is a knowledge of velocity at a given place and time. The shape, size and orientation of the orbit depend solely on the velocity; unless this is known with some precision the orbit is badly determined and no predictions of the future course of the vehicle are of any value. There are, of course, two different methods for obtaining these velocities; we can either difference successive positions of the vehicle, or we can integrate successive values of the accelerations. Both schemes are precisely the same in theory, but can give very different results in practice. The differencing scheme leads at once to one of the standard methods used for finding an orbit from three observations of one body at well-spaced intervals of time. Unfortunately the method is of little value if the apparent path of the observed body lies on a great circle, and this is almost certain to be the case with any observations of one planet made from a vehicle moving almost in the ecliptic.

However, a long series of observations, followed by a least-squares solution, would give continuously better results, but would still fall short of the ideal. If we assume with the authors that we can fix positions within 1000 miles, then velocities determined by differencing will be of the order of 1000 miles/day in error. It is true that the accuracy will increase with the number of fixes, but it is still not good enough to make predictions for months ahead. It may also be remarked that 1000 miles/day (roughly 0·01 mile/sec.) is hardly likely to be reached by any measures of stellar aberration. There is, perhaps, some promise in the idea of using the radio Doppler shift, but this is for the future, when there are many radio beacons, each with fully stabilized oscillators—and a stabilized receiver in the space vehicle.

The concept of an orbit, with its angular measures of node, anomaly, etc., is very much out of date, and is quite unnecessary. It lingers on in the literature because it affords a mental picture of the orbit in space, but a much more efficient method of representing the instantaneous orbit of a body in space is to quote its three coordinates and three velocity components at a given time. With these it is a simple matter to integrate the motion for future times, and this method is standard practice in dealing with comets and minor planets; the attractions of the Sun and planets along three axes are integrated to give velocities, and these integrated again to give positions. The process is precisely the same as that used for dealing with accelerometer readings, although the units may be different. If the initial data are sufficiently accurate the integration may be pushed as far ahead as is necessary, and it is quite usual for a complete ephemeris of a minor planet for a year ahead to be produced on a large digital computer in about 10 minutes.

The routine is, in fact, so simple that a large number of precomputed orbits could be prepared in a short time and stored on tape or film. If we combine these integration schemes with those obtained from the accelerometers, we have a complete representation of the path of the vehicle in space. Here, however, we come to a major difficulty, for the two parts of the scheme are of very different orders of magnitude. The methods for

dealing with the gravitational forces are well established and are not as difficult as the authors suggest. The forces are all quite small, and the largest of them—the attraction of the Sun—is only about $g/2000$ at the distance of the Earth. It would be quite sufficient, for an interplanetary journey of the type visualized in this paper, to have the attractions of the Sun, and Jupiter (and possibly Saturn) only, and to have these at an interval of ten days. Interpolation at this interval is simple; there is certainly no need for the interval of 25 seconds suggested by the authors, nor for the large number of planets mentioned.

The interval is not determined by the convenience of the user, but by the behaviour of the higher differences; this is not a problem in astronomy, but of normal computing methods. The tables which we use in ordinary computing are designed to allow interpolation with the minimum mental effort, but a table of sines to ten decimals, at an interval of 6°, may readily be interpolated if the user is willing to go out to eighth differences. A digital computer can do this with ease, but if the interval is too large the higher differences become difficult to handle, and may eventually be divergent, when interpolation is impossible.

It is for this reason that accelerometer readings are difficult to assimilate into the main scheme, for here the forces concerned are very large, and are applied for a very short time, so that the rate of change is enormous. It is not generally realised how large these thrusts can be. Thus, a force of 10 g applied for precisely one second will cause an increase in velocity of 218 mile/hr. (0·061 mile/sec.) and an increase in distance of over 5000 miles by the end of a day. And this does not take into account the fact that the size and shape of the orbit have been changed, and that the thrust, if not applied tangentially, will also change the whole orientation of the orbit in space. Such large and sudden forces could not possibly be entered into the integration scheme of the gravitational field; there must be two schemes with widely different intervals, and the two sets of results could then be added together by standard methods at fixed intervals. It is much more important that the accelerometers should be able to measure the three components of thrust continuously (rather than in steps) over very short time-ranges, and that they should be able to do so very accurately. A constant error of only 0·00001 g in the readings will give a velocity error of 455 mile/day after only one day, and a positional error of over two million miles after 100 days.

This suggests that no single method is sufficient. Perhaps a combination of the two will be useful—continuous fixes which can be used as a check on the integration schemes. But this is only part of the story. Suppose the navigator finds himself on the wrong course; what does he do then? We are told that he then calculates the changes in velocity which he must apply, but how is this done? It may be possible to derive algebraic equations which show that it *can* be done, but this is a very different matter, as any computer knows, from being able to do it. Quite frankly, I can think of no method, other than trial and error, by which the required three components of velocity can be calculated. Here is a case where a large number of precomputed orbits would be of value; they take up very little room, and it would not be difficult to devise a simple differential correction which would convert the present orbit into an improved one which would get the vehicle to about the right place at about the right time. The navigator then has to apply a thrust in a direction at a computed angle to the course of the ship for a limited time, but as he has only his (approximate) integration tables to give him that course, which cannot otherwise be measured, the correction also will be a mere approximation.

Let us face up to the fact that the methods of navigating in space that have been devised up to the present are about equivalent to those used by the old Elizabethan sailors; the navigator of a space vehicle in free fall will need just as much enterprise and must be just as willing to take risks as were the old sea-dogs of Drake's day. The solution to the problem is to evolve methods which do not demand frequent corrections of the course and here the ideal of constant drive is obviously the one at which to aim.

Yours faithfully,

J. G. PORTER.

Royal Greenwich Observatory,
Herstmonceux Castle,
Hailsham, Sussex.

REFERENCE

1. D. J. Cashmore and C. N. Gordon, *J.B.I.S.*, 1959–60, **17**, 46.

SIR,

We have read with interest the letters from Professor Lawden,[1] Dr. Noton,[2] and Dr. Porter[3] which comment on our paper.[4] Since some of the points made by these correspondents tend to overlap, we propose to reply to all three in this letter.

We are grateful to both Professor Lawden and Dr. Noton for having our attention called to the two excellent papers to which they refer.[5,6] There is undoubtedly a great deal to be said for not forcing the vehicle back on to the precalculated reference trajectory, especially when fuel is at a premium. The linear perturbation methods of correction set out in these two references are very similar, and it is interesting to note that they are extensions of a guidance system which could be used for ballistic missiles. The flexibility of such a system depends to some extent upon the range of position and velocity parameters over which the linear perturbation theory is accurate enough. This touches upon Professor Lawden's second point concerning the right balance between guidance accuracy and computation complexity. Perhaps a useful criterion would be that the initial guidance accuracy should be such that linear perturbation techniques could subsequently be used without significant error.

However, as Dr. Porter points out in the penultimate paragraph of his letter, the navigational process can only be one of successive approximation. The requirement on each phase of navigation are set by the next. The boost phase must be good enough for the linear perturbation theory to be sufficiently accurate in the mid-course phase. Similarly, the mid-course navigation and guidance must be accurate enough for the terminal guidance to become operative early enough with injection errors small enough for the vehicle to be able to correct.

We must agree with Dr. Noton's first point concerning radio guidance. In the early stages of space exploration, however, it is not realistic to expect radio tracking devices to be operative on the other planets, nor can we expect Earth-based radio systems to give adequate accuracy at very long ranges, for example take-off from a planet for a return journey. It seems wise therefore to make the navigator *primarily* self-contained, but able to make use of extra aids such as radio wherever it is available.

Dr. Noton may be right as to the approximation for computing V and D. This is basically a question of accuracy versus complexity of equipment and this can only be judged by a detailed systems assessment for each specific project.

The difficulty with stellar aberration is that even along the reference trajectory the velocity vector changes in magnitude and direction and therefore the distortion of the celestial sphere will vary with time. If the angular errors (perhaps as large as 1′ arc) are not acceptable during the periods that navigational corrections are being applied by the motors, a possible solution is to insert biases into the electrical outputs of the tracking telescopes. These biases might be precomputed on the basis of the reference trajectory, stored in the computer and changed as simple functions of time.

Again, Dr. Noton may be right on the question of the type of computer to be used. It is dangerous to be dogmatic at this early stage and only considerably more thought and experience can settle such matters. Dr. Porter's point concerning the different problems involved in the integration of thrust and gravitational accelerations is an example of a factor bearing on the nature of the computer. It would certainly appear that two separate integration processes are required here. In fact it would seem that a complete astro-intertial navigator feeding into a separate dead-reckoning computer is called for.

We were glad to see Dr. Porter dispensing with the classical concepts of orbits. The straightforward method of computing in x, y, z co-ordinates is perhaps less graphic but is certainly more suited to inertial navigation and computing.

We agree entirely with Dr. Porter that there is a real problem in determining velocity. This is perhaps the central problem in space navigation as we can foresee it today. At what point do position fixes give better velocity information than that available from dead reckoning? Clearly a very careful study must be made of the balance of payments between increasing the accuracy and frequency of planetary fixes and their reduction on the one hand and increasing the complexity and accuracy of the inertial navigator (and any other system) used in the boost phase and the dead reckoning computer on the other.

Yours truly,

D. J. CASHMORE.
C. N. GORDON.

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5 Swiftsgreen Road, Luton, Bedfordshire.

20 *April*, 1960.

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2. M. Noton, *ibid.*, 331.
3. J. G. Porter, *ibid.*, 382.
4. D. J. Cashmore and C. N. Gordon, *ibid.*, 46.
5. D. F. Lawden, *ibid.*, 1954, **13**, 215.
6. M. Noton, "Interplanetary Post-Injection Guidance," *Paper presented at A.R.S. Semi-Annual Meeting, San Diego, California*, **June, 1959.**

Meteoritic Dust and Rainfall

SIR,

On suggestion of Mr. Alexander Fucs, Chairman of the Associaçáo Brasileira de Astronomia (A.B.A.) I had the satisfaction to give a resumé before this Society in Rio de Janeiro of the excellent and very interesting paper "Meteoritic Dust and Ground Simulation of Impact on Space Vehicles" by Mr. D. H. Robey.[1] During our discussion of this paper were pointed out some essential points, that may perhaps interest you, the author and eventually also your readers. Therefore I have been charged to communicate them, as follows:

(1) As shown in the annexed Table I, extracted from my worldwide collection of daily rainfall figures, characteristic Brazilian Stations show rainfall peaks exactly on 17 December, 1955, corresponding to Bowen's Rainfall Correlation with the Leonid meteor shower. If allowance is made for a tolerance of 1 to 3 days of the rainfall effect from the exact date, as it appears in accordance with the mechanism involved (and as Bowen admits), then also the atmospheric precipitation peaks immediately following the exact date in Washington (U.S.A.), San Francisco (U.S.A.), Zürich-Kloten (Switzerland) and Sydney (Australia) may be attributed to the meteoritic dust of the Leonid meteor shower. The daily rainfall figures at the last December dates seem to be related with *Bielid I* meteor shower.

(2) Mr. Robey's new explanation of Bowen's Rainfall Correlation together with the alluded measurement of the density of micrometeorites in the high atmosphere by the Aerobee Rocket, confirm the sound fundament of Bowen's theory, on which I have developed the "Universal Meteor-Astronomical Laws of Atmospherical Precipitation."[2–3]

TABLE I.—*Some Examples of Daily Figures of Atmospheric Precipitation about the Date of 17 December, 1955*

December:	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
Rio de Janeiro D.F. (Jardin Botânico), Brazil	—	—	—	3·4	3·8	0·6	—	18·8	—	0·2	—	—	—	9·8	2·9	—	—	10·8	—	1·4	15·8	7·5	mm.
Fortaleza (Ceará), Brazil ..	—	—	—	—	2·4	—	1·2	2·0	—	—	—	—	—	—	2·0	1·0	1·0	—	—	1·0	2·4	—	mm.
Morro Velho (Minas Gerais), Brazil	—	—	—	22·9	3·0	23·6	8·9	40·6	25·4	18·0	0·3	0·3	5·6	15·0	10·9	1·5	19·1	39·9	5·1	49·0	35·1	43·2	mm.
Washington, U.S.A.	—	—	—	—	—	—	—	—	0·03	—	—	—	—	—	—	—	—	—	—	T*	—	—	in.
San Francisco (Cal.), U.S.A. ..	—	—	—	—	T	0·06	0·02	T	0·99	1·20	0·08	0·75	2·99	1·59	0·23	0·15	1·33	0·16	T	T	0·36	0·05	in.
Zürich-Kloten (Switzerland) ..	19	10	5	1	6	8	5	5	6	—	1	1	—	11	3	4	6	—	2	—	—	—	mm.
Sydney (Australia)	—	19	1	0	2	—	—	—	105	135	197	10	—	8	—	10	39	15	137	53	—	—	Points†

* T = Traces. † 1 point = 0·01 in. = 0·254 mm.

(3) Discussing already in 1956 in the World Meteorological Organization in Geneva the extremely world-wide spread of meteoritic condensation nuclei even to antipode stations to the direction of influx of the meteor streams, according to world-wide daily rainfall statistics, I suggested the Earth's gravitational attraction force to be responsible for this phenomenon. In the meantime the recent satellite experiments have shown in practice that objects in satellite orbits may revolve several times a day round the Earth. This, as well as Mr. Robey's conclusion (p. 29) that meteoroid dust could remain longer in satellite orbits, seem to confirm my above-mentioned suggestion.

(4) Mr. Robey mentions (in Table II, p. 29 of his paper) "ailergy" as a phenomenon in connection with freezing nuclei. We should be interested in any proved evidence of this kind.

Yours sincerely,

HUGO MELZER.

Icarai Palace Hotel, Praia das Flechas,
Niteroi, E. do Rio, Brazil.
10 July, 1959.

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2. H. Melzer, *Urania, Barcelona*, 1956, **41** (244), 191; *ibid.*, 1957, **42** (245), 117, 118.
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Electrical Theory of Gravity

SIR,

I wish to thank Mr. Cashmore and Mr. Gordon for bringing up some fundamental points in their letter.[1] The "practical" unit of electric charge is the coulomb, which belongs to the MKS system of units wherein the coulomb is defined, not by Coulomb's law, but in terms of the ampere, the unit of electrical current used in all scientific and engineering measurements.[2] As no test charge is involved in the definition of the ampere, the existing definition of the electric charge accepted above would need no remodelling to take account of the electrical theory of gravity presented in my paper.[3]

However, the coulomb may be expressed in terms of the statcoulomb, which is defined by Coulomb's law as as that charge which repels an equal charge of the same sign with a force of one dyne when the charges are separated by one centimetre. The precise relationship[2] is:

$$1 \text{ coulomb} = 2{\cdot}99592 \times 10^9 \text{ statcoulombs.}$$

Now let us consider what would happen to this relationship if we modified the definition of statcoulomb in terms of the attraction between two equal charges of opposite electrical sign. Then we find that this force of attraction would be 1 part in 10^{36} larger than the force of repulsion (between electrical charges of equal magnitude and like sign) to account for the electrical theory of gravity. This means that the "modified statcoulomb" (for unlike charges) should be 1 part in 10^{18} smaller than the present statcoulomb defined above. Or to put the other way around, if we were to remodel the definition of statcoulomb in terms of a force of attraction of one dyne between unlike charges separated by one centimetre, then the present statcoulomb would be 1·000 000 000 000 000 001 of this "modified statcoulomb." This figure, for all present precision experiments is the same as unity and the relationship between the "modified statcoulomb" and the coulomb would be

$$1 \text{ coulomb} = 2{\cdot}99592 \times 10^9 \text{ "modified statcoulombs"}$$

which is the same as that for the statcoulomb to the precision that we are concerned with here or even to the eighteenth place of decimals if ever we can achieve that degree of precision. Hence since there is so little difference between the "modified statcoulomb" and the present statcoulomb there would not be any necessity to remodel the definition of the statcoulomb either.

The unit electric field will also not be much affected by the electrical theory of gravity. The basic definition of unit electric field in the MKS system is that a force having a magnitude of 1 newton would be exerted on a test charge of 1 coulomb placed in the field at that point. If the source of the above electric field and the test charge are of opposite electrical sign then this force would be one part in 10^{36} larger than that if the source of the field and the test charge were of like sign (to account for gravity). Here again the difference is so small as to be negligible for all electrical measurements that in the interest of simplicity we may maintain the present definition of scalar magnitude of the unit electric field. The reason for trying to encourage simplicity is that the above definition is one of those to be first studied by a newcomer to electrical field theory and it would be imposing an additional burden for a newcomer to have to study two definitions instead of one, especially

as none of his electrical measurements would show up any difference between them.

Furthermore, maintenance of the present basic definition will cause no difficulty later in electromagnetic theory because in the calculation of the electric field in advanced problems, it is now the trend to compute it as the gradient of the potential

$$\vec{E} = -\text{ grad } V = \vec{i}\frac{\partial V}{\partial x} + \vec{j}\frac{\partial V}{\partial y} + \vec{k}\frac{\partial V}{\partial z}$$

e.g., as in Chapter 3 of Reference (2). In such a computation the test charge does not enter in the electric field and so there is no conflict with the electrical theory of gravity.

Finally it appears that though there is not a 1:1 correspondence between the unit of electric charge and the unit electric field, nevertheless because the electrical theory of gravity calls for a

1 : 1·000 000 000 000 000 000 000 000 000 000 000 000 01

correspondence which is so close to the 1:1 correspondence that for practical purposes it would be appropriate to keep the extensive structure of electromagnetic theory as it is and treat gravity as a special item under electromagnetic theory and/or present the electrical theory of gravity in the chapter on gravitation.

There could be very useful modifications of the "mechanisms" of electrical repulsions and electrical attractions so as to provide powerful propellent for spaceflight. But these would be the subject of patent applications and so for the time being would be outside the realm of pure electromagnetic theory.

Yours faithfully,

FREDDY BA HLI,
Director of Research.

Union of Burma Applied Research Institute,
Kanbe, Rangoon, Burma.

14 *June*, 1960.

REFERENCES

1. D. J. Cashmore and C. N. Gordon, *J.B.I.S.*, 1959–60, **17**, 217.
2. Francis Weston Sears. "Principles of Physics. II.—Electricity and Magnetism." **1946**: Cambridge, Mass. (Addison Wesley Press).
3. F. Ba Hli, *J.B.I.S.*, 1958–59, **16**, 540.

SIR,

Dr. Ba Hli's correspondence[1] contains gross errors in physics:

(1) The diagonal force is always smaller than the non-diagonal force in his Fig. 1, regardless of his assumption of attractive force being larger than the repulsive for within one part in 10^{36}. How can he justify a not *attractive* force?

(2) The diagonal electric force approaches zero as the separation of the charges of the dipole increases to infinity. So, the net electric repulsive, not attractive, force varies with the moment of the dipole.

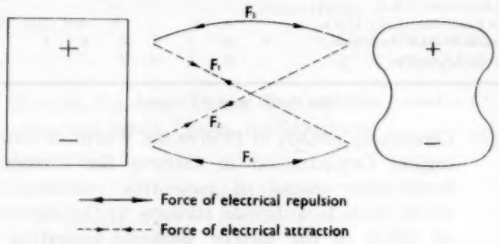

FIG. 1.

Dr. Ba Hli's net electric force is not only *not attractive* but it is merely the result of an electrostatic couple and bears no relation to the unique properties of the gravitational field, any more than the Lorentz force $\hat{v} \times \hat{B}$ of a rotating body *does not* bear any relation to the gravitational field (regardless of how similar they may be), as some have proposed otherwise. I strongly recommend that Dr. Ba Hli read Václav Hlavatý's book,[2] because he keeps referring to the gravitational field as a force.

E. OKRESS, JNR.

81–10 135th St., Kew Gardens 35,
New York, U.S.A.

21 *May*, 1960.

REFERENCES

1. F. Ba Hli, *J.B.I.S.*, 1957–58, **16**, 540.
2. Václav Hlavatý, "Geometry of Einstein's Unified Field Theory," **1957**: Groningen (P. Noordhoff Ltd.).

Range and Accuracy of Ballistic Missiles: A Correction

SIR,

I am sorry to say that a slight error has been found in my paper.[1] In Equation (11), p. 90, the radical should be preceded by a plus sign, rather than the minus sign shown. This does not affect any other equation, but Equation (11) is itself a useful final result, so the correction is worth while.

Yours truly,

IRVIN G. HENRY.

394 South Chester Avenue,
Pasadena 5, Calif., U.S.A.

1 *May*, 1960.

REFERENCE

1. Irvin G. Henry, *J.B.I.S.*, 1959–60, **17**, 88.

NEWS AND ANNOUNCEMENTS

B.I.S. NEWS

Alteration of the Bye-Laws: Officers of the Society

At its meeting on 11 June, Council resolved that the following Bye-Law be adopted and added to the existing Bye-Laws of the Society:

41. There shall be substituted as from 1 July, 1960, the titles of President and Vice-President for those of Chairman and Vice-Chairman respectively, provided that neither title shall be deemed to confer any additional powers or duties on the Officers concerned. The President for the time being shall exercise the functions of the Chairman as stated in the Articles of Association, and the provisions of the Bye-Laws with regard to Chairman and Vice-Chairman shall be read as if they referred to President and Vice-President respectively after 1 July, 1960.

This marks a return to the pre-war practice of the Society and also brings the structure of the Society more into line with that of other professional and scientific institutions. When the Society was incorporated under the Companies Act in 1945 no provision was made for the offices of President and Vice-President; in recent years Council has considered the matter on several occasions. It is now felt that the change is highly desirable and will be welcomed by members and by other organizations with which we are associated.

Dr. L. R. Shepherd thus becomes the first post-war President and the third since the Society was founded. It is particularly appropriate that he should now be the first to bear the title after its reintroduction, for as Chairman of Council in six of the last seven years he has directed our activities throughout a period of rapid growth and progress.

With the adoption of the new Bye-Law, Mr. K. W. Gatland and Dr. W. R. Maxwell become Vice-Presidents of the Society.

Scottish and Yorkshire Branches

A special meeting of the Council was held on Saturday, 16 July to discuss matters affecting branches and informal groups of the Society, and was attended by representatives from each United Kingdom branch.

At this meeting two of the items on the agenda were applications from the Scottish and Yorkshire provisional branches to be granted full branch status. Council decided unanimously to agree to both requests, and on behalf of the Society the President expressed congratulations to the officers and members of these branches and good wishes for the future.

Lecture Programme, Second Half-Session

A number of posters advertising the Society's lectures and symposia to be held during the second half of the 1960 session are now being printed. Members and others who have opportunities for displaying these on the notice boards of their works, colleges or offices, etc., may obtain them on application to the Secretary, 12 Bessborough Gardens, London, S.W.1.

Posters advertising meetings of Branches are also obtainable from the Branch Secretaries, whose names and addresses appear facing the inside of the front cover of this issue.

It is intended that in future these posters will bear the words

Further particulars may be obtained from . . .

followed by a space in which the person arranging the display can write his or her name and address, or department, etc.

Grant from Smithsonian Astrophysical Observatory

The Society has received a grant of £125 from the Smithsonian Astrophysical Observatory towards expenses incurred by Moonwatch teams during the year ended 30 June, 1960. This is in recognition of the useful contribution to satellite tracking being made by these teams.

There are three B.I.S. Moonwatch teams, in Scotland, the Isle of Wight, and Bristol. Their leaders are respectively: Dr. A. E. Roy, Department of Astronomy, The University, Glasgow, W.2; Mr. S. T. Smith, Pippins Cottage, Church Hill, Totland Bay, Isle of Wight; and Mr. J. Carter, 7 Hurlingham Road, Bristol. Members in these areas who are interested in participating in the work of the teams should write to the appropriate team leader.

A fourth Moonwatch team is in course of formation in the Midlands area and interested members are invited to write to Mr. J. D. Llewellyn, 80 West Green Drive, Stratford-on-Avon, Warwickshire.

Other teams will be established in centres where there are sufficient members to ensure regular manning of the observation posts. Those interested in setting up such a team should write to the Secretary.

Assistant Editor of the Journal

Mr. A. H. S. Candlin has been appointed to a vacancy on the Publications Committee arising from the resignation of Mr. D. J. Cashmore, on grounds of ill-health. Mr. Candlin will also act as Assistant Editor of the *Journal*.

Election of Members

The following elections were made at the Council Meeting on 11 June, 1960.

New Fellows.

JOHN ACKROYD, B.Sc., 272, Badminton Road, Downend, Bristol.
DOUGLAS ALLANSON, 64, Kings Hedges Road, Cambridge.
PETER BAUER, Leobeth, New Road, Barton, Cambridge.
CYRIL THOMAS BRADSHAW, A.M.I.MechE., A.F.R.Ae.S., 22, Locke King Road, Weybridge, Surrey.
J. FRANK CONEYBEAR, M.E., Laurel Hills Road, Rt. 6, Raleigh, N.C., U.S.A.
EDWIN GERARD DAVIDSON, Byways, Kingswood Road, Tylers Green, Buckinghamshire.
ALEXANDER GEORGE EVERARD, B.Sc., 73, New Beach Road, Darling Point, Sydney, Australia.
SIR ALFRED HUBERT ROY FEDDEN, M.B.E., D.Sc., Hon. F.R.Ae.S., Hon. F.I.A.S., Buckland Old Mill, Bwlch, Breconshire, South Wales.
STANLEY CHARLES GHOSE, M.B., Ch.B., M.A., Ph.D., D.Sc., M.I.E.E., 33, Heathfield Road, Acton, London, W.3.
WILLIAM ROBERT HAWKS, A.F.R.Ae.S., 114, Gurney Court Road, St. Albans, Hertfordshire.
EDWARD W. S. HULL, Rt. 1, Box 145, Springfield, Virginia, U.S.A.
NORMAN ERIC HYDE, 38, Priory Road, Gt. Malvern, Worcestershire.
HERBERT D. JENKINS, B.S., M.E., 2348 Loma Vista Place, Los Angeles, 39, California, U.S.A.
ERIC KORNGOLD, B.S., M.A., 9, Albion Place, Newton Centre, Mass., U.S.A.
HUA LIN, A.M.I.Mech.E., Sc.D., 1718E. 55 Street, Seattle 5, Washington, U.S.A.
ARTHUR FREDERICK MCLEAN, A.M.I.Mech.E., 1604 N. Chicago Street, South Bend, Indiana, U.S.A.
CHARLES ROBERT SINDEN MANDERS, M.A., Ph.D., 36, Chatsworth Gardens, West Harrow, Middlesex.
EDWIN JUSTIN MERRICK, M.E., McKean Road, Ambler, Penn., U.S.A.
GEOFFREY KEITH CHARLES PARDOE, B.Sc., D.I.C., 2, St. James Road, Harpenden, Hertfordshire.
ERIC WILLIAM RUNDLE, A.F.R.Ae.S., 152, Burbage Road, London, S.E.21.
ARTHUR HORACE SANDFORD, 6, Sandy Close, Hertford.
CLIFFORD ARTHUR SEYMOUR, A.M.I.Mech.E., The British Petroleum Co. Ltd., B.P. House, Ropemaker Street, London, E.C.2.
JAMES TAYLOR, M.A., Little Frimley, Portsmouth Road, Camberley, Surrey.
JACK ALFRED Thalhimer, B.S., 6870, Meadowbrook Boulevard, Minneapolis, 26, Minnesota, U.S.A.

Members elected to the Fellowship.

ROBERT WAUCHOPE BASS, M.A., Ph.D., Chief Scientist, Aerospace Division, Aeronca Mfg. Corp., Baltimore 28, Maryland, U.S.A.
THOMAS FRANK CHEANEY, B.S., c/o Broward Title Co., P.O. Box 1382, Ft. Lauderdale, Florida, U.S.A.
BARTON SEBRING PULLING, B.A., Idaho State College, Pocatello, Idaho, U.S.A.
RICHARD STANTON JONES, "Ruffins," Pallance Road, Northwood, Cowes, Isle of Wight.
WALTER EUGENE TENGELSEN, B.S.E.E., 59, Grist Mill Lane, Halesite, New York, U.S.A.

New Associate Fellows.

JOSEPH A. ALBERT, B.S.E.E., Navy BOQ, White Sands Missile Range, New Mexico, U.S.A.
ENRIQUE B. ASTURIAS, Yepocapa, Guatemala.
ROBERT BRUCE, "Yasme," Hillfield Road, Chalfont St. Peter, Buckinghamshire.
EDWARD DONALD CHRISTMAN, B.S., 41-07, 74 Street, Elmhurst 73, Queens New York, N.Y., U.S.A.

BASIL FRANCIS COLLINS, 72, Mt. Merrion Avenue, Belfast, 6, Northern Ireland.
MACON C. ELLIS, Jnr., B.S., Artillery Road, Yorktown, Va., U.S.A.
KENNETH HUGH FOULKES, 58, Westbourne Grove, Yeovil, Somerset.
ROBERT LAWRENCE GERVAIS, M.S., 1000, Granville Ave., West Los Angeles 49, California, U.S.A.
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ANTHONY JAMES LEGG, B.Sc., c/o D.S.I.R. Radio Research Station, University of Malaya, Singapore 10.
KEITH ALAN MARLOW, 52, Foxfield, Bandley Hill, Stevenage, Hertfordshire.
DAVID JOHN PALIN, M.A., LL.B., Avoca, St. Andrew's, Guernsey, Channel Islands.
BERNARD PAIEWONSKY, M.A., M.S.E., 65, Shadybrook Lane, Princeton, New Jersey, U.S.A.
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BARRY ALLEN STEIN, M.A., 4711, Bedford Avenue, Brooklyn 35, N.Y., U.S.A.
EDWIN LEE WHITE, B.E., 7, Angus Trail, N.W., Atlanta 5, Georgia, U.S.A.
RONALD WILKES, 9, Melrose Road, Weybridge, Surrey.
PAUL VAN ZYTVELD, B.S., 6067 N. Lucerne, Chicago 46, Ill., U.S.A.

Members elected to Associate Fellowship.

MICHAEL JAMES SCOTT BELTON, B.Sc., "Middlefield," Gainsborough, Lincolnshire.
AVERIL BURTON CHATFIELD, B.Sc., 5114, Newton Street, Torrance, California, U.S.A.
JOHN FAUST, A.B., 1516-10th Street, Berkeley 10, California, U.S.A.
JERRY LEE HARPER, M.S., General Delivery, Evergreen, Colorado, U.S.A.
CYRIL EDWARD SHEEMAN HORSFORD, B.A., Flat 4, 26, Medway Street, London, S.W.1.
DEAN RUTLEDGE HULING, B.A., 2420 E. 11th Avenue Denver 6, Colorado, U.S.A.
WILLIAM ALOYSIUS KINNEY, A.B., University Club, 1135 16th Street N.W., Washington 7, D.C., U.S.A.

Members elected to Senior Membership.

GEOFFREY BOTTOMLEY, 28, Merefield Street, Rochdale, Lancashire.
ARNULF WALTER ESTERER, 932, Norwood Drive, Ashtabula, Ohio, U.S.A.
ROBERT LIONEL FANTHORPE, 23, Norwich Road, Dereham, Norfolk.
GORDON CRESSWELL GEACH, 296, Teignmouth Road, Torquay, Devon.
RAYMOND ALBERT HARRAWAY, 38, Waldeck Street, Reading, Berkshire.
JOHN ALEXANDER MACLEOD, B.A., 4211, VanHorne Avenue, Montreal 26, Quebec, Canada.
ERIC CONRAD MILLER, 2, Helston Place, Abbots Langley, Hertfordshire.
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RONALD HUBERT SYDNEY SPACKMAN, "Log Cabin," Norwood Road, Milnerton, Cape Town, S. Africa.
WILFRED ARTHUR WHITEHOUSE, 2, Church Crescent, St. Albans, Hertfordshire.
JOHN NEVILLE WOOD, c/o Canadian Marconi Co., P.O. Box N. 400, St. Johns, Newfoundland, Canada.

New Members.

DONALD BABBAGE, 13, Russell Gardens, Ilford, Essex.
SIDNEY GEORGE BRIGHTON, 21, Grove Park, Colingdale, London, N.W.9.
GORDON WILLIAM BRUNT, Jeddah, Hook Hill, Freshwater, Isle of Wight.
JOHN GEORGE DAVID CARPENTER, B.Sc., 6, Sherriff Court, Sherriff Road, London, N.W.6.
JOHN PAUL DAVIS, B.S., 650, Branch Street, San Luis Obispo, Calif., U.S.A.
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MARTIN RICHARD EDMUNDS, 7, Palace View, Bromley, Kent.
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SOPHIE EMILIE HARRIS, 3, Park Road, Dartington, Totnes, Devon.
WILLIAM KENNETH HARTMANN, 1025, Manor Road, New Kensington, Pennsylvania, U.S.A.
LYNN MCNEILLE HASTIE, B.Sc., Christ College, Park Street, Hobart, Tasmania, Australia.
GEORGE ANDRE THEODORUS HEILLEGGER, c/o Radio Caribe, Nieuwestraat 22B, Curacao, Netherlands Antilles.
FREDERIC ROGER HEATHCOTE, 79, Reddings Road, Moseley, Birmingham 13.
JOHN JAMES HERRINGTON, 318, Hednesford Road, Norton Canes, Nr. Cannock, Staffordshire.
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ALAN DAVID LEVIN, M.S.Ae., 690 Victor Way, Apt. 2, Mountain View, California, U.S.A.
DAVID BERNARD LOVEMAN, 462, Palmer Avenue, Teaneck, New Jersey, U.S.A.
HUGH ROBERT LYLE, The White Cottage, Kingsley Green, Nr. Haslemere, Surrey.
GORDON OLIVER MOODY, New Zealand Electricity Department, Sub Station, Mataroa, New Zealand.
ANDREW FERGUSON NIMMO, 131, Wilton Street, Glasgow, N.W.
JOSEPH DESMOND PARRY, B.Sc., 10, Earl Street, Keighley, Yorkshire.
PERKSINS LEFEVRE PATTON, Route 1, Box 135, Clovis, New Mexico, U.S.A.
WILLIAM JOHN PERRY, 1A, Pendoylan Street, Newtown, Cardiff.
DAVID EUGENE PITTS, 1315 N.W. 21st, Oklahoma City 6, Oklahoma, U.S.A.
JOHN EDWARD PREEDY, 94, Greenview Avenue, Beckenham, Kent.
MICHAEL QUINTON, "Rudyard," Lakeside Road, Lymm, Cheshire.
JOHN THOMAS SMITH, 1593 Philadelphia Street, Indiana, Pennsylvania, U.S.A.
ALAN KLEVEN STEWART, 236, Chandler, Elmhurst, Illinois, U.S.A.
KEITH STOREY, 6, Thoresby House, Cavendish Road, Idle, Bradford, Yorkshire.
DAVID GEORGE STRATTON, B.A., Air B.P. Trading Ltd., Ropemaker Street, London, E.C.2.
FRANCIS JAMES TEBO, M.S., Ormlie Lodge, Thurso, Caithness, Scotland.
ALASTAIR DE TEISSIER-PREVOST, Officers' Mess, R.A.F. Valley, Anglesey, North Wales.
GEOFFREY ERNEST THOMPSON, M.A., Harlow Cottage, Otley Road, Harrogate, Yorkshire.
BENJAMIN VAN DER MERWE, 43, Hoffe Street, Kroonstad, Orange Free State, South Africa.
RENATO VESCO, Via Isonso 5–12, Genova, Italy.
DANIEL VINCENDON, 29, Boulevard Sucher, Paris, 16, France.
CHARLES BRIAN WEATHERHOGG, "Treveth," 35, Pine Road, Tividale, Dudley, Worcestershire.

JOINT ACTIVITIES

Symposium on Space Navigation

A one-day symposium on Navigation and the Early Exploration of the Moon is being organized jointly by the British Interplanetary Society and the Institute of Navigation. The provisional programme includes the following papers (some of the titles are tentative):

"The Demands of Nearly Parabolic Lunar Trajectories on Differential Correction and Perturlation Theory," by Professor Samuel Herrick (University of California, Los Angeles; *also* Principal Astrodynamicist, Aeronutronic, a Division of Ford Motor Company).

"Guidance of Space Vehicles by Radio Measurements and Command," by A. R. Maxwell Noton (University of Nottingham).

"Role of Inertial Equipment in Lunar Flights," by D. J. Cashmore (English Electric Aviation Ltd.).

"Comparison and Implementation of Rendezvous Methods," by Dr. G. R. Arthur (General Electric Company, U.S.A.).

"Fuel Requirements for Mid-Course Corrections during Return from Manned Lunar Flight," by H. L. Bloom (General Electric Company, U.S.A.).

"Surface Navigation on the Moon," by P. A. E. Stewart (Advanced Projects Group, Hawker Siddeley Aviation Ltd.).

"Summary Paper," by D. O. Fraser (English Electric Aviation Ltd.).

The Symposium will be held in the Lecture Theatre of the Royal Geographical Society, 1 Kensington Gore, London, S.W.7, on Friday, **18 November, 1960.** Members and others who wish to attend the Symposium should write to the Secretary of the Institute of Navigation at this address.

European Symposium on Space Technology

The first European Symposium on Space Technology is being held in London early in 1961. It is being organized by the Society, with the co-operation of the other principal European astronautical societies, and will take place in the Council Room of the Federation of British Industries, 21 Tothill Street, London, S.W.1, from Monday 26 June to Wednesday, 28 June, 1961, inclusive.

The Symposium is intended primarily to bring together scientists and engineers from Western Europe to discuss the possibility of formulating a joint programme of space research. A detailed programme will be given in a later issue of the *Journal*, but members and others are invited to register now (Registration Fee £5 5s. 0d.). Applications should be sent to The Secretary, British Interplanetary Society, 12 Bessborough Gardens, London, S.W.1.

Twelfth International Astronautical Congress

The Twelfth International Astronautical Congress will be held in New York early in October, 1961, probably immediately before the Annual Meeting of the American Rocket Society. It is hoped that it will be possible to hold the Congress in the United Nations Headquarters.

This will be the first I.A.F. Congress to be held outside Europe and should be a memorable occasion: it is possible that the arrangements may include a visit to the Cape Canaveral launching base. It is thought that over 3000 people will attend the meeting; to ensure that European participation is as large as possible, our American hosts intend to endeavour to arrange relatively inexpensive accommodation for those with limited means.

The fare from Europe will of course be the largest item in the cost of attending the Congress; if sufficient people decided to attend, it would be possible for a special plane to be chartered for the occasion. If this could be done well in advance the fare would be of the order of £90–£100 per person. Members and others who think there is a possibility that they might attend the Twelfth Congress should let the Secretary know as soon as possible.